K23U 3431

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2023
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
3B03MAT : Analytic Geometry and Applications of Derivatives

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark.

1. State Rolle's Theorem.
2. Find the focus and directrix of the parabola $y^2 = 12x$.
3. Find the asymptotes of the spiral $r = \frac{a}{\theta}$.
4. Find the equation of the tangent at any point (x, y) to the curve $x^{2/3} + y^{2/3} = a^{2/3}$.
5. Find the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$ parallel to the x-axis.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Determine all the critical points of the function $f(x) = 6x^2 - x^3$.
7. Find the directrix of the hyperbola $r = \frac{25}{10 + 10\cos\theta}$.
8. Find the asymptote of the curve $r = a \tan \theta$.
9. Using Maclaurin's series, expand tan x upto the term containing x^5.
10. Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$ and $r = 2 \sin \theta$.

11. Find $\lim_{x \to \pi/2} \frac{\sec x}{1+\tan x}$.

12. For the cardioid $r = a(1 - \cos \theta)$, prove that $\phi = \theta/2$.

13. For the curve $x = a(\cos t + \log \tan t/2)$, $y = a \sin t$, prove that the portion of the tangent between the curve and the x-axis is constant.

14. Find the intervals in which the function $f(x) = x^3 - 12x - 5$ is increasing and the intervals on which f is decreasing.

15. Show that the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$ represents a hyperbola.

16. Find the absolute maximum and minimum value of $f(x) = \frac{2}{3}x - 5$ on $[-2, 3]$.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Verify Rolle's theorem for $f(x) = (x + 2)^3 (x - 3)^4$ in $(-2, 3)$.

18. a) Find the focus and directrix of the parabola $x^2 = 6y$.

    b) Express the equation of the ellipse $16x^2 + 25y^2 = 400$ in the standard form.

19. Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

20. Find the angle of intersection of the curves $x^2 = 4y$ and $y^2 = 4x$.

21. Find the radius of curvature at the point $(3a/2, 3a/2)$ of the folium $x^3 + y^3 = 3axy$ on the curve $xy^2 = a^3 - x^3$.

22. a) Find the critical points of $f(x) = x^{1/3} (x - 4)$.

    b) Find the intervals in which the function f defined is increasing and decreasing.

23. Find a cartesian equation for the hyperbola centered at the origin that has a focus at $(3, 0)$ and the line $x = 1$ as the corresponding directrix.

PART – D

Answer **any two** questions. **Each** question carries **six** marks.

24. Find the equation of the normal at any point θ to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$. Verify that these normals touch a circle with its center at the origin and whose radius is constant.

25. a) Find the critical points of $f(x) = x^4 - 4x^3 + 10$.

    b) Find the intervals in which f defined is increasing and decreasing.

    c) Find the intervals where the graph of f is concave up and concave down.

    d) Sketch the general shape of f.

26. a) The ellipse $\left(\frac{x^2}{16}\right)+\left(\frac{y^2}{9}\right)=1$ is shifted 4 units to the right and 3 units up. Find the equation of the new ellipse in standard form.

    b) Find the foci, vertices and center of the new ellipse.

    c) Plot the new ellipse mentioned in a) part.

    d) Find the ellipse's equation in standard form with foci $\left(\pm\sqrt{2},0\right)$ and vertices $(\pm 2, 0)$.

27. Show that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a\cos\theta/2$.