K24U 1742

Reg. No.: ..............................

Name : ..................................

**Second Semester B.Sc. AI and ML Degree (CBCSS – OBE-Regular) Examination, April 2024**
**(2023 Admission Onwards)**
**Complementary Elective Course**
**2C02MAT – AIML : INTEGRATION AND LINEAR ALGEBRA**

Time : 3 Hours Max. Marks : 40

PART – A
**(Short Answer)**

Answer **all** questions from this Part. **Each** question carries 1 mark. **(6×1=6)**

1. Find $Lt_{x \to 0, y \to 0} \frac{xy^2}{x^2 + y^2 + 2}$.
2. Let $z = x^3 + y^3 - 3axy$. Find $\frac{\partial z}{\partial x}$.
3. Write the reduction formula for $\int \tan^n x dx$.
4. Define vector space.
5. Define characteristic polynomial of a matrix A.
6. When can you say that a matrix is diagonalizable ?

PART – B
**(Short Essay)**

Answer **any six** questions from this Part. **Each** question carries **2** marks. **(6×2=12)**

7. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$. Find $\frac{\partial^2 u}{\partial x \partial y}$.
8. If x increases at the rate of 2 cm per sec at the instant when x = 3 cm and y = 1 cm, at what rate must y be changing in order that the function $2xy - 3x^2y$ shall be neither increasing nor decreasing ?

P.T.O.

9. Find $\int \sin^3 x dx$.

10. Find $\int \cos^2 x dx$.

11. Define Basis of a vector space and give an example.

12. Define linear transformation. Give an example of a linear transformation from $R^2$ to $R^3$.

13. When can you say that a quadratic form is negative definite ?

14. State Cayley-Hamilton Theorem.

PART – C
**(Essay)**

Answer **any four** questions from this Part. **Each** question carries **3** marks. **(4×3=12)**

15. If $\theta = t^n e^{\frac{-r^2}{4t}}$, what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$ ?

16. If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$.

17. Evaluate $\int_0^a \frac{x^7}{\sqrt{(a^2 - x^2)}} dx$.

18. Evaluate $I_n = \int_0^a (a^2 - x^2)^n dx$ where n is a positive integer. Hence show that $I_n = \frac{2n}{2n+1} a^2 I_{n-1}$.

19. Prove that a square matrix A and its transpose $A^T$ have the same characteristic roots.

20. Find a linearly independent eigenvectors of the matrix $\begin{pmatrix} 2 & 7 \\ 6 & -9 \end{pmatrix}$ and diagonalize it.

## PART – D
## (Long Essay)

Answer **any two** questions from this Part. **Each** question carries **5** marks. **(2×5=10)**

21. Transform the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates.

22. Derive the formula for $\int \sin^p x \cos^q x dx$ where p and q are positive integers.

23. Give a set of three vectors in $R^3$ that are linearly dependent. Justify your answer.

24. Find the characteristic roots of the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and then verify Cayley Hamilton theorem. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.