K21U 0129

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – Reg./Supple./Improve.)
Examination, April 2021
(2014 – 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B12MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **1** mark.

1. Define an analytic function and give an example of a function which is not analytic at $z = -i$.
2. State Morera's theorem.
3. Define circle of convergence of a series.
4. $f(z) = \dfrac{1}{(z^2-1)(z+1)}$ has a pole at $z = -1$ of order __________ .

SECTION – B

Answer **any eight** questions, **each** question carries **2** marks.

5. If $\dfrac{8+3i}{9-2i} = a + bi$, then find a and b.
6. Sketch the set of points in the complex plane given by $|z - i| \leq 2$.
7. Solve the equation $e^z = i$.
8. Find the principal value of i^i.
9. Write the real and imaginary parts of cos z.

P.T.O.

10. State Cauchy's integral theorem.

11. Evaluate $\int_C \text{Re}(z)\,dz$, where C is the straight line joining z = 0 to z = 1 + 2i.

12. Evaluate $\oint_C \frac{z^2+1}{z^2-1}dz$, where C : |z – 1| = 1 (counter-clockwise).

13. Show that the series $\sum_{n=0}^{\infty} \frac{(3+4i)^n}{n!}$ is convergent.

14. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z-3i)^n$.

15. Define conditionally convergent series and give an example.

16. Find the Laurent series of $\frac{\sin z}{z^5}$ with centre 0.

17. Show that the sequence $\{z_n\}$ with $z_n = \left(1-\frac{1}{n^2}\right)+\left(2+\frac{2}{n}\right)i$ is convergent.

18. Define meromorphic function and give an example.

19. Find the zeros of the function $f(z) = (1 - z^4)^2$.

20. Find the residue of $f(z) = \frac{9z+i}{z(z-i)}$ at z = i.

SECTION – C

Answer **any four** questions, **each** question carries **4** marks.

21. Show that $f(z) = \bar{z}$ is nowhere differentiable.

22. Show that an analytic function of constant absolute value is constant.

23. Evaluate $\oint_C (z-z_0)^m dz$, where C is a circle of radius ρ with centre z_0 in counter-clockwise direction.

24. State and prove Liouville's theorem.

25. Show that the geometric series $\sum_{n=0}^{\infty} z^n$ converges, if |z| < 1.

26. Find a Maclaurin series of $f(z) = \tan^{-1} z$.

27. Show that $f(z) = e^{1/z}$ has an essential singularity at $z = 0$.

28. Show that zeros of analytic function $f(z) \neq 0$ are isolated.

SECTION – D

Answer **any two** questions, **each** question carries **6** marks.

29. a) State and prove Cauchy-Riemann equations.

    b) Show by using Cauchy-Riemann equations, $f(z) = z^3$ is analytic everywhere.

30. a) Find a conjugate harmonic function of $u = x^2 - y^2 - y$.

    b) Find $\frac{d}{dz}(l n\, z) = \frac{1}{z}$, where z is not a negative real or zero.

31. a) State and prove Cauchy's inequality.

    b) Evaluate $\oint_C \frac{z^4 - 3z^2 + 6}{(z+i)^3} dz$, where $C : |z + i| = 3$ (counter-clockwise).

32. a) If f(z) is analytic in a simply connected domain D, show that the integral of f(z) is independent of paths in D.

    b) Evaluate $\oint_C \frac{\tan z}{z^2 - 1}$, where $C : |z| = 3/2$ (counter-clockwise).

33. a) State Taylor's theorem.

    b) Find the Taylor series expansion of $f(z) = \frac{2z^2 + 9z + 5}{z^3 + z^2 - 8z - 12}$ at $z = 1$.

34. a) State Residue theorem.

    b) Evaluate $\oint_C \frac{z - 23}{z^2 - 4z - 5}$, where $C : |z - 2| = 4$ (counter-clockwise).
