K20U 1532

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2020
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B05MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

(Answer **all** the questions. **Each** carries **1** mark)

1. Find all real x so that $|x - 1| < |x|$.

2. Give two divergent sequences (x_n) and (y_n) such that $(x_n + y_n)$ is convergent.

3. State n^{th} term test.

4. Show that $f(x) = \frac{1}{x}, \forall x$ is not uniformly continuous on $(0, \infty)$. **(4×1=4)**

SECTION – B

(Answer **any eight** questions. **Each** carries **2** marks)

5. There does not exists a rational number r such that $r^2 = 2$. Prove.

6. For positive real numbers a and b, show that $\sqrt{ab} \leq \frac{1}{2}(a + b)$, where equality occurring if and only if a = b.

7. Define infimum of a set. Find inf S if $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$.

8. A sequence in $\mathbb{R}$ can have atmost one limit. Prove.

9. Prove that every Cauchy sequence is bounded.

10. If Σx_n and Σy_n are convergent, show that the series $\Sigma(x_n + y_n)$ is convergent.

11. Check the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$.

12. If $X = (x_n)$ is a decreasing sequence of real numbers with $\lim x_n = 0$, and if the partial sums (s_n) of Σy_n are bounded, prove that the series $\Sigma x_n y_n$ is convergent.

13. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that the set $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.

14. Give an example to show that every uniformly continuous functions are not Lipschitz functions. **(8×2=16)**

SECTION – C

(Answer **any four** questions. **Each** carries **4** marks)

15. State and prove Archimedean property of $\mathbb{R}$.

16. If S is a subset of $\mathbb{R}$ that contains at least two points and has the property

    If $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$.

    Show that S is an interval.

17. For $C > 0$, show that $\lim\left(C^{1/n}\right) = 1$.

18. Discuss the convergence of the Geometric series $\sum_{n=0}^{\infty} r^n$ for $r \in \mathbb{R}$.

19. If Σx_n is an absolutely convergent series in $\mathbb{R}$, show that any rearrangement Σy_k of Σx_n converges to the same value.

20. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that f is uniformly continuous on I. **(4×4=16)**

SECTION – D

(Answer **any two** questions. **Each** carries **6** marks)

21. a) Prove the existence of a real number x such that $x^2 = 2$.

    b) If $a, b \in \mathbb{R}$, show that $\left| |a| - |b| \right| \leq |a - b|$.

22. a) State and prove Bolzano Weierstrass Theorem for sequences.

    b) If $X = (x_n)$ is a bounded increasing sequence in $\mathbb{R}$, show that it converges and $\lim(x_n) = \sup\{x_n : n \in \mathbb{N}\}$.

23. a) State and prove D'Alembert ratio test.

    b) Check the convergence of the series whose n^{th} term is $\frac{(n!)^2}{(2n)!}$.

24. a) Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. If $\varepsilon > 0$, then there exists step functions $s_\varepsilon : I \to \mathbb{R}$ such that $|f(x) - s_\varepsilon(x)| < \varepsilon, \forall x \in I$.

    b) Let $f(x) = x$, $\forall x \in [0, 1]$. Calculate the first few Bernstein polynomials for f.

(2×6=12)
