K17P 1595

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (Regular) Examination, October 2017
(2017 Admission)
MATHEMATICS
MAT 1C02 : Linear Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Describe explicitly the linear transformation T from F^2 into F^2 such that $T\varepsilon_1 = (a, b)$, $T\varepsilon_2 = (c, d)$.

2. Let F be the subfield of the complex numbers and T be the function from F^3 into F^3 defined as $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$, if (a, b, c) is a vector in F^3. What are the conditions on a, b, c so that (a, b, c) is in the range of T ?

3. Let T be the unique linear operator on $\mathbb{C}^3$ for which $T\varepsilon_1 = (1, 0, i)$, $T\varepsilon_2 = (0, 1, 1)$, $T\varepsilon_3 = (i, 1, 0)$ is T invertible.

4. Let A and B be n×n matrices over the field F. Prove that if (I-AB) is invertible the I-BA is invertible and $(I-BA)^{-1} = I + B(I-AB)^{-1}A$.

5. Let V be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$ which are continuous. Let T be the linear operator on V defined by $(Tf)(x) = \int_0^x f(t)\,dt$. Prove that T has no characteristic values.

6. Prove that an orthogonal set of non zero vectors in an inner product space is linearly independent.

P.T.O.

PART – B

Answer **4** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – 1

7. a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose V is finite dimensional then prove that rank (T) + nullity(T) = dim V.

   b) Let V be an n-dimensional vector space over the field F and let W be an m-dimensional vector space over the field F then prove that the space L(V, W) is finite dimensional and has dimension mn.

8. a) Let g, $f_1, \ldots, f_r$ be linear functional on a vector space V with respective null spaces N, $N_1, N_2, \ldots, N_r$ then prove that g is a linear combination of $f_1, \ldots, f_r$ iff N contains the intersection $N \cap N_1 \cap N_2 \cap \ldots \cap N_r$.

   b) Let V and W be finite dimensional vector spaces over the field F. Let B be an ordered basis for V with dual basis B* and let B′ be an ordered basis for W with dual basis B′*. Let T be a linear transformation from V into W; let A be the matrix of T relative to B, B′ and let B be the matrix of T^t relative to (B′*, B*). Then prove that $B_{ij}=A_{ji}$.

9. a) Let V be a finite dimensional vector space over the field F and let $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same field F and let $\beta_1, \beta_2, \ldots, \beta_n$ be any vector in W. Then prove that there is precisely one linear transformation T form V into W such $T\alpha_j = \beta_j$, j=1,2 ,..... ,n.

   b) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. If T is invertible then prove that the inverse function T^{-1} is a linear transformation from W onto V.

Unit – 2

10. a) Let T be a linear operator on the finite dimensional space V. Let $c_1, c_2, \ldots, c_k$ be the distinct characteristic vector of T and let W_i be the space of characteristic vector associated with the characteristic value c_i. If $W = W_1 + W_2 + \ldots\ldots\ldots + W_k$, then prove that $\dim W = \dim W_1 + \dim W_2 + \ldots\ldots + \dim W_k$.

b) Let the linear operator on $\mathbb{R}^3$ which is representation the standard ordered basis by the matrix.

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$ check whether T is diagonalizable or not.

11. a) Let V be a finite dimensional vector space over the field F. Let F be a commuting family of triangulable linear operators on V. Then prove that there exist an ordered basis for V such that every operator in F is represented by a triangular matrix in that basis .

b) Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is diagonalizable iff the minimal polynomial for T has the form $P= (x - c_1)(x - c_2) .... (x - c_k)$.

12. Let T be a linear operator on a finite dimensional vector space V. If f is the characteristic polynomial for T, then prove that f(T) = 0.

Unit – 3

13. a) Let T be a linear operator on a finite dimensional space V. If T is diagonalizable and $c_1, c_2, \ldots, c_k$ be the distinct characteristic vector of T then prove that there exist linear operators $E_1, E_2, ..., E_k$ on V such that

i) $E_iE_j = 0, i \neq j$

ii) E_i is a projection

iii) The range of E_i is the characteristic space for T associated with c_i.

b) State and prove Primary decomposition theorem.

14. State and prove cyclic decomposition theorem .

15. a) Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is a Idempotent linear transformation of V onto W, $W^{\perp}$ is the null space of E and $V=W \oplus W^{\perp}$.

b) Prove that every finite dimensional inner product space has an orthonormal basis.