K17P 1594

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (Regular) Examination, October 2017
(2017 Admission)
MATHEMATICS
MAT1C01 : Basic Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Let R be a finite commutative ring with unity. Show that every prime ideal is maximal.
2. Find the product of the polynomials $f(x) = 4x - 5$ and $g(x) = 2x^2 - 4x + 2$ in $\mathbb{Z}_8[x]$.
3. Show that a subgroup of a group G having index 2 is a normal subgroup of G.
4. Is {(2, 1), (3, 1)} a basis for $\mathbb{Z} \times \mathbb{Z}$? Prove your assertion.
5. Prove that no group of order 20 is simple.
6. Is $\phi : \mathbb{Z}_{12} \longrightarrow \mathbb{Z}_3$ be a homomorphism such that $\phi(1) = 2$. Find Ker (ϕ).

PART – B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Define group action. Let X be a G-set and let $x \in X$. Then prove that $|Gx| = (G : G_x)$.

   b) Prove that direct product of abelian groups is abelian.

P.T.O.

8. a) Show that every group of order 255 is cyclic.

   b) Find all abelian groups (up to isomorphism) of order $2^3 \times 3 \times 5^2$.

9. a) State and prove Cauchy's Theorem for groups.

   b) If m divides the order of a finite abelian group, then prove that G has a subgroup of order m.

Unit – II

10. a) Let D be an integral domain and let $S = \{(a, b) | a, b \in D, b \neq 0\}$. Show that the relation on S defined by $(a, b) \sim (c, d)$, if and only if $ad = bc$ is an equivalence relation.

    b) Let F be the collection of equivalence classes of the equivalence relation on $S = \{(a, b) \mid a, b \in D, b \neq 0\}$, D is an integral domain, defined by $(a, b) \sim (c, d)$, if and only if $ad = bc$. For $[(a, b)]$ and $[(c, d)]$ in F, prove the equations $[(a, b)] + [(c, d)] = [(ad + bc, bd)]$ and $[(a, b)]\,[(c, d)] = [(ac, bd)]$ are a well defined operation on F.

11. a) Let H be a subgroup of G and let N be a normal subgroup of G. Then prove that $HN/N \simeq H/(H \cap N)$.

    b) State Schreier theorem. Explain with an example.

12. a) Let $G \neq 0$ be a free abelian group with a finite basis. Then prove that every basis of G is finite and all bases of G have the same number of elements.

    b) Show that a free abelian group contains no nonzero elements of finite order.

Unit – III

13. a) State Eisenstein criterion and prove that the polynomial $\phi_p(x) = x^{p-1} + x^{p-2} + ... + x + 1$ is irreducible over $\mathbb{Q}$ for any prime p.

    b) Let $\phi : R \longrightarrow R'$ be a ring homomorphism and let N be an ideal of R and N′ be an ideal of R′.

       i) Prove that $\phi(N)$ is an ideal of $\phi(R)$

       ii) Prove that $\phi^{-1}(N')$ is an ideal of R

       iii) Give an example of an ideal N such that $\phi(N)$ need not be an ideal of R′.

14. a) Prove that an ideal $\langle p(x) \rangle \neq \{0\}$ of F[x] is maximal if and only if p(x) is irreducible.

b) i) Show that a factor ring of a field is either the trivial ring of one element or is isomorphic to the field.

ii) Show that if R is ring with unity and N is an ideal such that $N \neq R$, then R/N is ring with unity.

15. a) If G is a finite subgroup of the multiplicative group $\langle F^*, \cdot \rangle$ of a field F, then prove that G is cyclic.

b) Prove that if F is a field, every proper non trivial prime ideal of F[x] is maximal.