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K19P 1190

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS-Reg./Suppl./Imp.) Examination, October - 2019
(2017 Admn. Onwards)
MATHEMATICS
MAT 3E 01 : GRAPH THEORY

Time : 3 Hours Max. Marks : 80

Instructions to Candidate:

1) Answer any **4** questions from Part - A. Each question carries **4** marks.
2) Answer any **4** questions without omitting any units from Part - B. Each question carries **16** marks.

PART - A

I. Answer any **Four** questions. Each question carries **Four** marks.

1) Draw a 4-chromatic graph containing no triangles.

2) Prove that a set $S \subseteq V$ is an independent set of G iff V\S is a covering of G.

3) Show that the Peterson graph is 4-edge chromatic.

4) If G is a plane graph then prove that $\sum_{f \in F} d(f) = 2\varepsilon$.

5) Prove that a simple graph G is n-edge connected if and only if given any pair of distinct vertices U and V of G, then are at least n edge disjoint paths from U to V.

6) Let U and V be two distinct vectrus of a graph G. Then prove that a set F of edges of G is U-V separating if and only if every U-V path has at least one edge belonging to F.

P.T.O.

PART - B

Answer any **4** questions without omitting any Unit. Each question carries **16** marks.

UNIT - I

II. a) Define the edge independence number and edge covering number of a graph G and prove that if minimum degree of G is greater than zero, then sum of edge covering number and edge independence number is equal to the number of vertices. **(8)**

b) For any two integers $k \geq 2$ and $l \geq 2$ prove that $r(k, l) \leq r(k, l-1) + r(k-1, l)$. **(8)**

III. a) If G is 4-chromatic, then prove that G contains a subdivision of k_4. **(8)**

b) If G is a connected simple graph and is neither an odd cycle nor a complete graph then prove that $x \leq \Delta$. **(8)**

IV. a) In a bipartite graph G with $\delta > 0$ prove that the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering. **(8)**

b) Define a (k, l) Ramsey graph, give one example of a Ramsay graph and show that $r(k,l) \leq \binom{k+l-2}{k-1}$. **(8)**

UNIT - II

V. a) Prove that every planar graph is 5 vertex colourable. **(8)**

b) Let G be a non planar connected graph that contain no subdivisions of k_5 or k_{33} and has a few edges as possible, then prove that G is simple and 3 connected. **(8)**

VI. a) State and prove Eulers formula for planar graph and deduce that $k_{3,3}$ in non planar. **(8)**

b) Show that inner bridges avoid one another. **(8)**

VII. Prove that a graph is planar if and only if it contains no subdivisions of k_5 or k_{33} further check k_{33}-c is planar or not. **(16)**

UNIT - III

VIII. a) Show that a matching M in G is a maximum matching if and only if G contains no M any menting path. **(12)**

b) When will you say that a graph G is factorable give example of a graph G, which have 3 factors. **(4)**

IX. Prove that a graph G has a perfect matching if any only if $O(G - S) \leq |S|$ for all $S \subset V$. **(16)**

X. a) State and prove the max-flow-min-cut Theorem. **(8)**

b) Let u and V be two vertices of a graph G then prove that the maximum number of edge disjoint U-V paths in G equals the minimum number of edges is a U-V separating set. **(8)**

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