0151063 **K19P 1188**

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS-Reg./Suppl./Imp.) Examination, October - 2019

(2017 Admission Onwards)

Mathematics

MAT 3C13 : COMPLEX FUNCTION THEORY

Time : 3 Hours Max. Marks : 80

PART - A

Answer any **Four** questions. Each question carries **4** marks. **(4×4=16)**

1. Prove that an elliptic function without poles is a constant.
2. Derive the Legendre's relation $\eta_1\omega_2 - \eta_2\omega_1 = 2\pi i$.
3. Can an analytic function on an arbitrary region be expressed as the limit of a sequence of polynomials? Justify your claim.
4. Define the terms function element, germ and analytic continuation along a path.
5. Let G be an open subset of C. If $u : G \rightarrow C$ is harmonic, prove that u is infinitely differentiable.
6. Define a subharmonic function. Also show that every harmonic function is subharmonic.

PART - B

Answer any **Four** questions without omitting any unit. Each question carries **16** marks. **(4×16=64)**

UNIT - I

7. a) Define the period module of a function f(z) which is meromorphic in the whole plane.

P.T.O.

b) Prove that there exists a basis (w_1, w_2) such that the ratio $\tau = w_2 / w_1$ satisfies the following conditions:

i) $\text{Im } \tau > 0$

ii) $-\frac{1}{2} < \text{Re}\,\tau \le \frac{1}{2}$

iii) $|\tau| \ge 1$

iv) $\text{Re}\,\tau \ge 0$ if $|\tau| = 1$

Show further that the ratio τ is uniquely determined by these conditions, and there is a choice of two, four, or six corresponding bases.

8. a) Prove that the zero $a_1, \ldots, a_n$ and poles $b_1, \ldots, b_n$ of an elliptic function satisfy $a_1 + \ldots + a_n \equiv b_1 + \ldots + b_n \pmod M$.

b) With usual motations, prove that the weierstrass P - function satisfies the differential equation $P'(z)^2 = 4\,P(z)^3 - g_2\,P(z) - g_3$.

9. a) Define Riemann zeta function $\zeta(z)$ and prove that for Re z > 1,

$$\zeta(z)\,\Gamma(z) = \int_0^\infty \left(e^t - 1\right)^{-1} t^{3-1} dt.$$

b) State and prove Euler's theorem.

c) State the Riemann Hypothesis.

UNIT - II

10. State and prove Runge's theorem.

11. a) Let G be an open connected subset of . $\mathbb{C}$ If G is simply connected, prove that n(r, a) = 0 for every closed rectifiable curve r in G and every point a in $\mathbb{C} - G$.

b) State and prove Mittag - Leffler's theorem.

12. a) State Schwarz reflection principle.

b) State and prove the monodromy theorem.

c) Let (f, D) be a function element which admits unrestricted continuation in a simply connected region G. Prove that there is an analytic function $F : G \to \mathbb{C}$ such that F(z) = f(z) for all z in D.

UNIT - III

13. a) State and prove the mean value theorem for harmonic functions.

b) Let G be a region and let u and v be continuous real valued functions on G that have the MVP. If for each point a in the extended boundary $\partial_\infty G$, $\limsup_{z \to a} u(z) \le \liminf_{z \to a} v(z)$, then prove that either u(z) < v(z) for all z in G or u = v.

c) State the minimum principle for harmonic functions.

14. a) Define the poisson kernel $P_r(\theta)$. Prove that

i) $\int_{-\pi}^{\pi} P_r(\theta) = 2\pi$

ii) $P_r(\theta) > 0$ for all θ

b) Let D = {z : |z|<1} and suppose that $f : \partial D \to \mathbb{R}$ is a continuous function. Prove that there is a unique continuous function $u D^- \to \mathbb{R}$ such that

i) u(z) = f(z) for z in ∂D;

ii) u is harmonic in D.

15. a) State and prove Harnack's theorem

b) Derive Jensen's formula.
