K18P 1036

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (Reg.) Examination, October 2018
MATHEMATICS
(2017 Admn. Onwards)
MAT3E01 : Graph Theory

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **any four** questions from Part – **A**. **Each** question carries **4** marks.*

*2) Answer **any 4** questions without omitting **any** Unit from Part – **B**. **Each** question carries **16** marks.*

PART – A

I. Answer **any 4** questions. **Each** question carries **4** marks.

1) Define a (k, l) Ramsay graph and give one example.

2) In a critical graph, prove that no vertex cut is a clique.

3) For a bipartite graph G, show that $\chi'(G) = \Delta$.

4) If G is a planar graph then prove that every subgraph of G is planar.

5) Let l be a flexible vertex labelling of G. If G_l contains a perfect matching M^*, then prove that M^* is an optimal matching of G.

6) Let u and v be two distinct vertices of a graph G. Then prove that a set S of vertices of G is u – v separating if and only if every u – v path has at least one internal vertex belonging to S.

PART – B

Answer **any 4** questions without omitting any unit. **Each** question carries **16** marks.

Unit – I

II. a) Define the independence number and covering number of a graph and prove that the sum of the independence number and covering number is the number of vertices. 8

b) Define the Ramsay number r(k, l) and show that $r(k, k) \geq 2^{k/2}$. 8

P.T.O.

III. a) If a simple graph G contain no K_{m+1}, then prove that G is degree majorised by same complete m-partite graph H. Also show that if G has the same degree sequence as H then $G \cong H$. 8

b) Define the chromatic number χ (G) of a graph G. Give example of a critical graph and a graph which is not critical. Also for a graph G, show that $\chi \leq \Delta + 1$. Give an example of a graph where $\chi = \Delta + 1$. 8

IV. a) For any positive integer k, prove that there exist a k-chromatic graph containing no triangles. 8

b) If G is 4-chromatic, then prove that G contain a subdivision of k_4. 8

Unit – II

V. a) If G is bipartite show that $\chi' = \Delta$. 5

b) Let G be a connected graph that is not an odd cycle, then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two. 6

c) What is a time tabling problem and explain how one can solve the time tabling problem using edge colouring ? 5

VI. a) Define a dual graph of a graph G and prove or disprove – "Dual of isomorphic plane graph are isomorphic". 6

b) If G is a connected plane graph, then prove that $V - \Sigma + \phi = 2$, further deduce that K_5 is non planar. 10

VII. State and prove Kuratowski's theorem. 16

Unit – III

VIII. a) In a bipartite graph, prove that the number of edges in a maximum matching is equal to the number of vertices in a maximum covering. 10

b) If G is a k-regular bipartite graph with k > 0 then prove that G has a perfect matching. 6

IX. a) Prove that every 3-regular graph without cut edge has a perfect matching. 6

b) Explain in detail the Hungarian method to find an M-augmenting path in a graph and draw its flow-chart. 10

X. a) Let f be a flow on a network N = (V, A) and let f have value d. If $A(X, \overline{X})$ is a cut in N then prove that $d = f(X, \overline{X}) - f(\overline{X}, X)$. Also prove that $d \leq C(X, \overline{X})$. 8

b) State and prove Mengers theorem. 8