K18P 1034

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree (Reg.) Examination, October 2018
MATHEMATICS
(2017 Admn. Onwards)
MAT3C13 : Complex Function Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions. **Each** question carries **4** marks.

1. Prove that an elliptic function without poles is a constant.
2. Define the Weierstrass sigma function $\sigma(z)$ and show that it is an odd function.
3. Find a meromorphic function in the plane with a pole at every integer.
4. Suppose that f(z) is analytic in a region G which is symmetric with respect to the real axis and f(x) is real for all x in $G \cap \mathbb{R}$. Prove that $f(z) = \overline{f(\overline{z})}$ for all z in G.
5. If u is harmonic, show that $f = u_x - iu_y$ is analytic.
6. Prove or disprove : every harmonic function is subharmonic. **(4×4=16)**

PART – B

Answer **any four** questions **without** omitting any **unit**. **Each** question carries **16** marks.

Unit – I

7. a) Prove that a discrete module consists either of zero alone, of the integral multiples nw of a single complex number $w \neq 0$, or of all linear combinations $n_1w_1 + n_2w_2$ with integral coefficients of two numbers w_1 and w_2 with nonreal ratio w_2/w_1.

   b) Prove that any two bases of the same period module are connected by a unimodular transformation.

P.T.O.

8. a) Describe the construction of the Weierstrass P-function.

   b) Prove that addition theorem for the P-function :

$$P(z+u) = -P(z) - P(u) + \frac{1}{4}\left(\frac{P'(z)-P'(u)}{P(z)-P(u)}\right)^2$$

9. a) Define the Riemann zeta function $\zeta(z)$. Prove that for Rez > 1, $\zeta(z)$

$$\Gamma(z) = \int_0^\infty (e^t - 1)^{-1} t^{z-1}\, dt.$$

   b) Derive Riemann's functional equation $\zeta(z) = 2(2\pi)^{z-1}\Gamma(1-z)\ \zeta(1-z)$ $\sin\left(\frac{1}{2}\pi z\right)$ for $-1 < \text{Re} z < 0$.

Unit – II

10. a) Let K be a compact subset of the region G. Prove that there are straight line segments $r_1, ..., r_n$ in G – K such that for every function f in H(G),

$$f(z) = \sum_{k=1}^{n} \frac{1}{2\pi i} \int_{r_k} \frac{f(w)}{w-z}\, dw$$ for all z in K and the line segments form a finite number of closed polygons.

   b) Let G be an open connected subset of $\mathbb{C}$. If n(r, a) = 0 for every closed rectifiable curve r in G and every point a in $\mathbb{C}$– G, then prove that $\mathbb{C}_\infty$ – G is connected.

11. a) State and prove Mittag-Leffler's theorem.

   b) Define analytic continuation along a path.

12. a) State and prove Schwarz reflection principle.

   b) With usual assumptions, what is the meaning of saying that a function element (f, D) admits unrestricted analytic continuation in G ?

   c) State monodromy theorem.

Unit – III

13. a) State and prove the mean value theorem for harmonic functions.

b) Let $D = \{z : |z| < 1\}$ and suppose that $f : \partial D \to \mathbb{R}$ is a continuous function. Prove that there is a unique continuous function $u : D^{-} \to \mathbb{R}$ such that :

i) $u(z) = f(z)$ for all z in ∂D and

ii) $u(z)$ is harmonic in D.

14. a) If $u : G \to \mathbb{R}$ is a continuous function which has the mean value property, prove that u is harmonic.

b) State and prove Harnack's theorem.

15. a) Let G be a region and $f : \partial_{\infty} G \to \mathbb{R}$ a continuous function. Prove that

$u(z) = \sup \{\varphi(z) : \varphi \in P(f, G)\}$ defines G harmonic function u on G.

b) Derive Jensen's formula. **(4×16=64)**