K18P 1035

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree (Reg.) Examination, October 2018
MATHEMATICS
(2017 Admn. Onwards)
MAT3C14 : Advanced Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this part. **Each** question carries **4** marks.

1. Give an example of a sequence of functions which converges pointwise but not uniformly.
2. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, prove that $\{f_n + g_n\}$ converges uniformly on E.
3. Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2x}$. On what intervals does it fail to converge uniformly ?
4. Show that e^x defined on $\mathbb{R}^1$ satisfy the relation $(e^x)' = e^x$.
5. Define orthogonal system of functions and give an example.
6. Prove that $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$. **(4×4=16)**

PART – B

Answer **4** questions from this part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) If $\{f_n\}$ is a sequence of continuous function on E, and if $f_n \to f$ uniformly on E, then show that f is continuous on E.

   b) If $f_n \in \mathcal{R}(\alpha)$ on [a, b] and if $f(x) = \sum_{n=1}^{\infty} f_n(x)$ $(a \le x \le b)$, the series converging uniformly on [a, b], then prove that $\int_a^b f d\alpha = \sum_{n=1}^{\infty}\int_a^b f_n d\alpha$.

P.T.O.

8. a) Even if $\{f_n\}$ is a uniformly bounded sequence of continuous functions on a compact set E, prove that there need not exist a subsequence which converges pointwise on E.

   b) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E, then prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in E$.

9. State and prove Stone Weierstrass theorem.

Unit – II

10. a) Suppose $\sum c_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n (-1 < x < 1)$. Then prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

    b) Define analytic functions and give an example.

11. a) Suppose the series $\sum a_n x^n$ and $\sum b_n x^n$ converge in the segment $S = (-R, R)$. Let E be the set of all $x \in S$ at which $\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n$. If E has a limit point in S, then prove that $a_n = b_n$ for n = 0, 1, 2, ... Hence $\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n$ for all $x \in S$.

    b) Let $\{\phi_n\}$ be orthonormal on [a, b]. Let $s_n(x) = \sum_{m=1}^{n} c_m \phi_m(x)$ be the n[th] partial sum of the Fourier series of f, and suppose $t_n(x) = \sum_{m=1}^{n} \gamma_m \phi_m(x)$. Then prove that $\int_a^b |f - s_n|^2 dx \le \int_a^b |f - t_n|^2 dx$, and equality holds if and only if $\gamma_m = c_m (m = 1, 2, \ldots, n)$.

12. a) If, for some x, there are constants $\delta > 0$ and $M < \infty$ such that

    $|f(x+t) - f(x)| \le M|t|$ for all $t \in (-\delta, \delta)$, then prove that $\lim_{N \to \infty} S_N(f; x) = f(x)$.

    b) If f(x) = 0 for all x in some segment J, then prove that $\lim S_N(f; x) = 0$ for every $x \in J$.

    c) If f is continuous (with period 2π) and if $\epsilon > 0$, then prove there is a trigonometric polynomial P such that $|P(x) - f(x)| < \epsilon$ for all real x.

Unit – III

13. a) Define the dimension of a vector space and give an example.

    b) Define basis of a vector space.

    c) Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then prove that dim $X \leq r$.

14. a) Suppose E is an open set in $\mathbb{R}^n$, f maps E into $\mathbb{R}^m$, f is differentiable at $x_0 \in E$, g maps an open set containing f(E) into $\mathbb{R}^k$, and g is differentiable at $f(x_0)$. Then prove that the mapping F of E into $\mathbb{R}^k$ defined by $F(x) = g(f(x))$ is differentiable at x_0 and $F'(x_0) = g'(f(x_0)f'(x_0))$.

    b) Suppose f maps a convex open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$, f is differentiable in E, and there is a real number M such that $\| f'(x) \| \leq M$ for every $x \in E$. Then prove that $| f(b) - f(a) | \leq M | b - a |$ for all $a \in E$, $b \in E$.

15. State and prove implicit function theorem. **(4×16=64)**