K18P 0232

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (Regular) Examination, March 2018
MATHEMATICS
(2017 Admn.)
MAT2C10 – Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Eliminate the arbitrary function F from $F\left(x+y, x-\sqrt{z}\right)=0$ and find the corresponding Partial differential equation.

2. Find the general integral of the PDE : x (y – z) p + y (z – x) q = z (x – y).

3. Prove that the solution to the Dirichlet problem is stable.

4. Prove that the solution to the Dirichlet problem, if it exist, is unique.

5. Convert the initial value problem : $y'' + \lambda y = f(x)$, $y(0) = 1$, $y'(0) = 0$ into an integral equation.

6. Solve the integral equation $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$ by iterative method.

(4×4=16)

PART – B

Answer **four** questions from this Part, without omitting **any** Unit. **Each** question carries **16** marks.

Unit – 1

7. a) Prove that the Pfaffian differential equation :
$X \cdot dr = P(x, y, z)dx + Q(x, y, z)\, dy + R(x, y, z)\, dz = 0$ is integrable if and only if $X \cdot \text{curl}(X) = 0$.

   b) Prove that the Pfaffian differential equation :
$(1 + yz)\, dx + z(z - x)\, dy - (1 + xy)\, dz = 0$ is integrable and find the corresponding integral.

P.T.O.

8. a) Explain Charpit's method to find a complete integral of a first order partial differential equation in two independent variables.

   b) Find a complete integral of the PDE : $z^2 - pqxy = 0$.

9. a) Explain the method to find the solution of a semi-linear equation by the method of characteristic curves.

   b) Find the integral surface passing through the initial data curve $x = 1$, $z = y^2 + y$ of the equation $x^3 z_x + y(3x^2 + y) z_y = z(2x^2 + y)$.

Unit – 2

10. a) Derive the equation governing the transverse vibrations of an infinite string.

    b) Derive d' Alembert's solution of wave equation.

11. a) State and prove maximum principle.

    b) Reduce the equation $y^2 u_{xx} - 2xy u_{xy} + x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y$, to a canonical form and solve it.

12. a) What is Dirichlet problem for the upper half plane. Using convolution theorem prove that the solution to the Dirichlet problem for the upper half plane is $u(x, y) = \frac{y}{x} \int_{-\infty}^{\infty} \frac{f(\xi)}{y^2 + (x - \xi)^2} d\xi$.

    b) Using part (a) to find the solution of the Neumann problem for the upper half plane.

Unit – 3

13. a) Convert $y'' - \sin xy' + e^x y = x$ with initial conditions $y(0) = 1$, $y'(0) = -1$ to a Volterra integral equation of the second kind. Conversely, derive the original differential equation with the initial condition from the integral equation obtained.

    b) Solve the homogeneous Fredholm integral equation of the second kind $y(x) = \lambda \int_0^{2\pi} \sin(x + \xi) y(\xi) d\xi$.

14. a) Find the Green's function of the boundary value problem :
$y'' = 0$, $y(0) = y(l) = 0$.

b) Let $\Gamma(x, \xi, \lambda)$ be the resolvent (or reciprocal) kernel for the Fredholm integral equation then prove that the resolvent kernel satisfies the integral equation :

$$y(x) = K(x, \xi) + \lambda \int_0^x K(x, \xi)\, y(\xi)\, d\xi$$

15. a) Obtain an approximate solution of the integral equation
$y(x) = \int_0^1 \sin(x\xi)\, y(\xi)\, d\xi + x^2$ by replacing $\sin(x\xi)$ by the first two terms of its power series : $\sin(x\xi) = x\xi - \frac{x\xi}{3!} + \dots$

b) Find the iterated kernel for the kernel $K(x, \xi) = \sin(x - 2\xi)$, $0 \le x \le 2\pi$, $0 \le \xi \le 2\pi$. **(4×16=64)**