K18P 0230

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (Regular) Examination, March 2018
Mathematics
(2017 Admn.)
MAT 2C08 : ADVANCED TOPOLOGY

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Give an example of a bounded metric space that is not compact. Justify your example.
2. Is a compact subset of a topological space necessarily closed ? Justify your answer.
3. Prove that every subspace of a T_2-space is a T_2-space.
4. Let X ={1, 2, 3} and $\mathcal{J}$ = {ϕ, {1}, {2}, {1, 2}, X}. Determine whether (X, $\mathcal{J}$) is a normal space.
5. Show that there is a homomorphism h : $\mathbb{R} \to (-1, 1)$.
6. Show that real line $\mathbb{R}$ with usual topology is contractible. (4×4=16)

PART – B

Answer **any four** questions from this part without omitting of **any** Unit. **Each** question carries **16** marks.

UNIT – I

7. a) Define :
   i) Bolzano-Weierstrass property.
   ii) Countable compactness.
   iii) T_1-space.

   b) Let(X, $\mathcal{J}$) be a T_1-space. Prove that X is countably compact if and only if it has the Bolzano Weierstrass property.

P.T.O.

8. a) Prove that a topological space $(X, \mathcal{T})$ is compact if and only if every family of closed subsets of X with the finite intersection property has a nonempty intersection.
   b) Prove that the product of any finite number of compact spaces is compact.

9. a) Define a locally compact space. Show that the real line with usual topology is locally compact but not compact.
   b) Prove that every closed subspace of a locally compact Hausdorff space is locally compact.
   c) Show that the continuous image of a locally compact space need not be locally compact.

UNIT – II

10. a) Let $(X, \mathcal{T})$ be a topological space. Prove that $(X, \mathcal{T})$ is a T_1-space if and only if for each $x \in X$, $\{x\}$ is closed.
    b) Let $\{(X_\alpha, \mathcal{T}_\alpha) : \alpha \in \Lambda\}$ be a family of topological spaces and let $X = \prod_{\alpha \in \Lambda} X_\alpha$.
    Prove that $(X, \mathcal{T})$ is regular if and only if $(X_\alpha, \mathcal{T}_\alpha)$ is regular for each $\alpha \in \Lambda$.

11. a) Let $(X, \leq)$ be a well ordered set and let $\mathcal{T}$ be the order topology on X. Prove that $(X, \mathcal{T})$ is a normal space.
    b) Prove that a T_1-space is completely normal if and only if every subspace of it is normal.

12. a) Prove that every second countable space is Lindelof. Show by an example that a Lindelof space need not be second countable.
    b) Prove that every regular Lindelof space is normal.

UNIT – III

13. a) State (no proof) Urysohn's lemma. Deduce that every normal space is completely regular.
    b) Prove that a T_1-space $(X, \mathcal{T})$ is normal if and only if whenever A is a closed subset of X and $f : A \to [-1, 1]$ is a continuous function, then there is a continuous function $F : X \to [-1, 1]$ such that $F|_A = f$.

14. State and prove Alexander subbase theorem.

15. a) Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces. Prove that the relation being homotopic is an equivalence relation on the collection $C(X, Y)$ of all continuous functions that map X into Y.
    b) Let $(X, \mathcal{T})$ be a topological space, let $x_0 \in X$ and let $[\alpha], [\beta], [\gamma] \in \Pi_1(X, x_0)$. Prove that $([\alpha] \circ [\beta]) \circ [\gamma] = [\alpha] \circ ([\beta] \circ [\gamma])$.

(4×16=64)

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