Reg. No. : ....

Name : ....

K20P 0348

II Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.) Examination, April 2020
(2017 Admission Onwards)
MATHEMATICS
MAT2C08 – Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this part. **Each** question carries **4** marks : **(4×4=16)**

1. Give an example, with proper reasoning, of a bounded metric space that is not compact.
2. Is a continuous function from a compact metric space to a metric space always uniformly continuous ? Justify your answer.
3. Show that regularity is a topological property.
4. Is the topological space (X, T) normal, where X = {1, 2, 3, 4} and T = {ϕ, {1}, {1, 2}, {1, 2, 3}, X} ? Justify your answer.
5. Show that a T_1 space, which can be imbedded as a subspace of I^ω, is a separable metric space.
6. Let (X_n, d_n) be a metric space for each $n \in \mathbb{N}$ and let $X = \Pi_{n \in \mathbb{N}} X_n$. Prove that $d(x, y) = \sum_{n=1}^{\infty} \frac{d_n(x_n, y_n)}{2^n}$ for $x, y \in X$ is a metric on X.

PART – B

Answer **any four** questions from this part without omitting **any** unit.
Each question carries **16** marks : **(4×16=64)**

Unit – I

7. a) Prove that every open cover of a metric space with the Bolzano-Weierstrass property has a Lebesgue number.

   b) Prove that a metric space is compact if and only if it is complete and totally bounded.

P.T.O.

8. a) Show that the product of two compact spaces is compact.
   b) Show that compactness is a topological property.
   c) Give an example, with proper reasoning, of a compact set that is not closed.

9. a) Prove that every closed subspace of a locally compact Hausdorff space is locally compact.
   b) Let X be a locally compact space. If there is an open continuous function from X onto Y, then show that Y is locally compact.
   c) Give an example, with proper reasoning of a compact set that is not sequentially compact.

Unit – II

10. a) Give an example, with proper reasoning, of a T_1 space that is not T_2.
    b) Let X be a topological space and Y a Hausdorff space. If $f : X \to Y$ is continuous, then prove that $\{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is a closed set.
    c) Prove that a T_1 space is regular if and only if for each $p \in X$ and each neighbourhood U of p, there is a neighbourhood V of p such that $\overline{V} \subseteq U$.

11. a) Let $\{(X_\alpha, T_\alpha) : \alpha \in \wedge\}$: be a family of topological spaces with $X = \Pi_{\alpha \in \wedge} X_\alpha$. Prove that (X, T) is regular if and only if (X_α, T_α) is regular for each $\alpha \in \wedge$.
    b) Let $(X, \leq)$ be a well-ordered set and let T denote the order topology on X. Prove that (X, T) is a normal space.

12. a) Prove that a T_1 space is completely normal if and only if each of its subspace is normal.
    b) Prove that every regular Lindelof space is normal.

Unit – III

13. a) State and prove Urysohn's Lemma.
    b) Prove that the set of dyadic numbers in I is dense in I.

14. a) State and prove Tychonoff theorem.
    b) Prove that, if (X, T) is a T_1, regular and second countable space, then X can be imbedded as a subspace of I^ω.
    c) Show that the space I^ω is mertrizable.

15. a) For two spaces (X, T). (Y, U), show that the relation defined by $f \simeq g$ if f is homotopic to g is an equivalence relation on C(X, Y).
    b) Let (X, T) be a topological space and $x_0 \in X$. Prove that the operation $\circ$ defined on $\pi_1(X, x_0)$ by $[\alpha] \circ [\beta] = [\alpha * \beta]$ is associative.
