K19P 0360

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (Reg./Suppl./Imp.) Examination, April 2019
(2017 Admission Onwards)
MATHEMATICS
MAT 2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Eliminate the arbitrary function F from $z = xy + F(x^2 + y^2)$ and find the corresponding partial differential equation.

2. Find the complete integral of $zpq = p^2(p^2 + xq) + q^2(q^2 + yp)$.

3. Prove that the solution of the Neumann problem is unique up to the addition of a constant.

4. Prove that the solution to the Dirichlet problem, if it exist, is unique.

5. Convert the initial value problem : $y'' + y = 0, y(0) = 0, y'(0) = 0$ into an integral equation.

6. Find the solution of the integral equation $g(x) = x + \int_0^1 x\xi^2 g(\xi)d\xi$. **(4×4=16)**

PART – B

Answer **four** questions from this Part, without omitting **any** Unit. **Each** question carries **16** marks.

Unit – 1

7. a) Prove that the system of equations $f(x, y, z, p, q) = 0$, $g(x, y, z, p, q) = 0$ are compatible if and only if $[f, g] = 0$.

   b) Prove that the equations

   $f = xp - yq - x = 0,$

   $g = x^2p + q - xz = 0$

   are compatible and find a one parameter family of common solutions.

P.T.O.

8. a) Explain Charpit's method to find a complete integral of a first order partial differential equation in two independent variables.

   b) Find a complete integral of $(p^2 + q^2)y - qz = 0$.

9. a) Explain the method to find the solution of a quasilinear equation by the method of characteristic curves.

   b) Solve the initial value problem for the quasilinear equation $zz_x + z_y = 1$ containing the initial data curve $C : x_0 = s, y_0 = s, z_0 = s/2$ for $0 \leq s \leq 1$.

Unit – 2

10. a) Prove that the solution of the following problem, if it exists, is unique.

    $y_{tt} - c^2 y_{xx} = F(x, t), 0 < x < \ell, t > 0,$

    $y(x, 0) = f(x), 0 \leq x \leq \ell,$

    $y_t(x, 0) = g(x), 0 \leq x \leq \ell,$

    $y(0, t) = y(\ell, t), t > 0$

    b) Find the solution of the above problem by the method of separation of variables when $F = 0$.

11. a) State and prove maximum principle.

    b) Solve the following heat conduction problem by the method of separation of variables.

    $u_t - ku_{xx} = 0 \; 0 < x < \ell, t > 0,$

    $u(0, t) = u(\ell, t) = 0, t > 0,$

    $u(x, 0) = f(x), 0 \leq x \leq \ell$

12. a) What is Dirichlet problem for the upper half plane ? Using Convolution theorem prove that the solution of the Dirichelet problem for the upper half plane is $u(x, y) = \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{f(\xi)}{y^2 + (x - \xi)^2} d\xi$.

    b) Prove that solution to the following Cauchy problem is not stable.

    $u_{xx} + u_{yy} = 0,$

    $u(x, 0) = 0,$

    $u_y(x, 0) = \frac{\sin nx}{n}$

Unit – 3

13. a) Prove that the linear differential equation :

$$y'' + a_1(x)y' + a_2(x)y = F(x)$$

with initial conditions $y(0) = C_0$ and $y'(0) = C_1$ can be transformed into non-homogeneous Volterra integral equation of the second kind.

b) Find the eigenvalues and the corresponding eigenfunctions of the integral equation.

$$y(x) = \lambda\int_0^1 (2x\xi - 4x^2)y(\xi)\,d\xi$$

14. a) Using Green's function, solve the boundary value problem :

$$y'' + y = x,\ y(0) = y(\pi/2) = 0.$$

b) Show that the integral equation $y(x) = 1 + \frac{1}{\pi}\int_0^{2\pi} \sin(x+\xi)y(\xi)d\xi$ possess infinitely many solutions.

15. a) Find the iterated Kernels $K_2(x, \xi)$ and $K_3(x, \xi)$ associated with $K(x, \xi) = |x - \xi|$ in the interval [0, 1].

b) Solve the integral equation :

$y(x) = 1 + \lambda\int_0^1 (1 - 3x\xi)y(\xi)d\xi$ by iterative method. **(4×16=64)**
