K19P 0358

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (Reg./Suppl./Imp.) Examination, April 2019
(2017 Admission Onwards)
MATHEMATICS
MAT2C08 : Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Show by an example that a bounded metric space need not be totally bounded.

2. Let A be a subset of a topological space $(X, \mathcal{T})$. If A is compact prove that every open cover of A by members of $\mathcal{T}_A$ has a finite subcover.

3. Give an example of a T_0-space that is not aT_1 space.

4. Prove that every closed subset of a normal space is normal.

5. Show that there is a homeomorphism $h : \mathbb{R} \to (-1, 1)$.

6. Let $(X, \mathcal{T})$ be a topological space and let f, g : X → I be continuous functions. Prove that f is homotopic of g. **(4×4=16)**

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

UNIT – I

7. a) Prove that a metric space having Bolzano-Weierstrass property is totally bounded.

   b) Let $(X, \mathcal{T})$ be a T_1 - space. Prove that X is countably compact if and only if it has the Bolzano-Weierstrass property.

P.T.O.

8. a) Prove that every compact subset of a Hausdorff space is closed.

   b) Prove that compactness is a topological property.

   c) Prove that a topological space $(X, \mathcal{T})$ is compact if and only if every family of closed subsets of X with the finite intersection property has a nonempty intersection.

9. a) When is a topological space $(X, \mathcal{T})$ said to be locally compact at a point p in X ? If $(X, \mathcal{T})$ is a Hausdorff space prove that X is locally compact at p if and only if there is a neighborhood U of p such that $\bar{U}$ is compact.

   b) Show that the continuous image of a locally compact space need not be locally compact.

   c) Prove that local compactness is preserved under open continuous functions.

UNIT – II

10. a) Let $(X, \mathcal{T})$ be a topological space. Prove that $(X, \mathcal{T})$ is a T_1-space if and only if for each $x \in X$, $\{x\}$ is closed.

    b) Prove that a T_1-space $(X, \mathcal{T})$ is regular if and only if for each member p of X and each neighborhood p of U, there is a neighborhood V of p such that $\bar{V} \subseteq U$.

    c) Prove that every subspace of a regular space is regular.

11. a) Let $\{(X_\alpha, \mathcal{T}_\alpha) : \alpha \in \Lambda\}$ be a family of topological spaces and let $X = \prod_{\alpha \in \Lambda} X_\alpha$.

    Prove that $(X, \mathcal{T})$ is regular if and only if $(X_\alpha, \mathcal{T}_\alpha)$ is regular for each $\alpha \in \Lambda$.

    b) Define a completely normal space. Prove that a T_1 space $(X, \mathcal{T})$ is completely normal if and only if every subspace of it is normal.

12. a) Let $(X, \leq)$ be a well ordered set, and let $\mathcal{T}$ denote the order topology on X. Prove that $(X, \mathcal{T})$ is a normal space.

    b) Prove that every second countable regular space is normal.

UNIT – III

13. a) State (no proof) Urysohn's lemma. Deduce that every normal space is completely regular.

    b) Prove that a T_1–space $(X, \mathcal{T})$ is normal if and only if whenever A is a closed subset of X and $f : A \rightarrow [-1, 1]$ is a continuous function, then there is a continuous function $F : X \rightarrow [-1\ 1]$ such that $F|_A = f$.

14. a) State (no proof) Alexander subbase theorem. Use it to prove that product of compact spaces is compact.

    b) For each $n \in \mathbb{N}$, let (X_n, d_n) be a metric space, let $X = \prod_{n \in \mathbb{N}} X_n$, and let $\mathcal{T}$ be the product topology on X. Prove that $(X, \mathcal{T})$ is metrizable.

15. a) State and prove Urysohn's metrization theorem.

    b) Let $(X, \mathcal{T})$ be a topological space, let $x_0 \in X$, and let $[\alpha] \in \Pi_1(X, x_0)$. Prove that there is $[\bar{\alpha}] \in \Pi_1(X, x_0)$ such that $[\alpha] \circ [\bar{\alpha}] = [\bar{\alpha}]\,[\alpha] = [e]$, where [e] is the identity element of $\Pi_1(X, x_0)$. **(4×16=64)**
