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Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS - Reg./Suppl./Imp.)
Examination, October - 2019
(2017 Admission onwards)
MATHEMATICS
MAT1C03 : REAL ANALYSIS

Time : 3 Hours Max. Marks : 80

Instructions to Candidate :

Answer any **four** questions from part A. Each question carries **4** marks. Answer any **four** questions from part B without omitting any unit. Each question carries **16** marks.

PART-A

1. For $x, y \in \mathbb{R}^1$, define $d(x,y) = \frac{|x-y|}{1+|x-y|}$. Determine whether d is a metric on $\mathbb{R}^1$.

2. Define discontinuity of the second kind and illustrate with an example.

3. If $c_0 + \frac{c_1}{2} + ... + \frac{c_{n-1}}{n} + \frac{c_n}{n+1} = 0$, where $c_0, c_1, ..., c_n$ are constants, prove that the equation $c_0 + c_1x + ... + c_{n-1}x^{n-1} + c_nx^n = 0$ has at least one real root between 0 and 1.

4. Suppose f is a bounded function on [a,b] and f^2 is Riemann integrable on[a,b]. Doesn't follow that f is Riemann integrable on [a,b]? Why?

5. If $f, g \in R(\alpha)$ on [a,b], prove that $f\, g \in R(\alpha)$.

6. Determine whether the function f defined by $f(x) = x\cos(\pi/2x)$ *if* $x \neq 0$, f(0)=0 is of bounded variation on [0,1].

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PART-B

UNIT-I

7. a) Prove that every infinite subset of a countable set is countable.

   b) Prove that the set of all sequences whose elements are the digits 0 and 1 is uncountable.

   c) Let X be a metric space and $k \subset y \subset x$. Prove that k is compact relative to x if and only if k is compact relative to y.

8. a) Define a perfect set. Prove that a nonempty perfect set in $\mathbb{R}^k$ is uncountable.

   b) Prove that a subset E of $\mathbb{R}^1$ is connected if and only if it has the following property:

   If $x \in E, y \in E$, and $x < z < y$, then $z \in E$.

9. a) Prove that a mapping f of a metric space X into a metric space Y is continuous if and only if f^{-1} (V) is open in X for every open set V in Y.

   b) Let f be a continuous mapping of a compact metric space into a metric space Y. Prove that f(x) is compact.

   c) Let f be monotonic on (a,b). Prove that the set of point of (a,b) at which f is discontinuous is at most countable.

UNIT-II

10. a) Let f be defined on [a,b]. If f has a local maximum at a point $x \in (a,b)$, and if $f'(x)$ exists, prove that $f'(x) = 0$

    b) State and prove Taylor's theorem.

11. a) State L' Hospital's rule and show that it fails to hold for complex valued functions.

    b) Define the Riemann-Stieltjes integral of f with respect to α over [a,b]. How is this integral related to the Riemann integral of f on [a,b] ?

    c) If f is monotonic on [a,b] and α is continuous on [a,b] , prove that $f \in R(\alpha)$.

12. a) If $f_1, f_2 \in R(\alpha)$ on [a,b], prove that $f_1+f_2 \in R(\alpha)$ and that $\int_a^b (f_1+f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$.

b) Let f be a bounded real function on [a,b], α increases monotonically and $\alpha' \in R$ on [a,b]. Prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$ on [a,b] and $\int_a^b f\, d\alpha = \int_a^b f(x)\, \alpha'(x) dx$.

UNIT-III

13. a) Let $f \in R$ on [a,b]. For $a \le x \le b$, define $F(x) = \int_a^x f(t)\, dt$. Prove that F is continuous on [a,b].

b) State and prove the fundamental theorem of calculus.

c) When is a function f said to be of bounded variation on [a,b] ? If f is continuous on [a,b] and if f' exists and is bounded on (a,b), i.e $|f'(x)| \le A$ for all $x \in (a,b)$, prove that f is of bounded variation on[a,b].

14. a) If f is of bounded variation on [a,b], prove that f is bounded on [a,b]. Is the converse true ? Justify your claim.

b) Let f be of bounded variation on [a,b]. Define V by $V_f(a,x)$ if a $a < x \le b$ and V(a) = 0. prove that every point of continuity of f is also a point of continuity of V. Also prove that the converse is true.

15. a) Let f be of bounded variation on [a,b] and let $c \in (a,b)$. prove that f is of bounded variation on [a,c] and [c,b], and $V_f(a,b) = V_f(a,c) + V_f(a,b)$.

b) Let f be of bounded variation on [a,b]. Let V be defined on [a,b], by $V(x) = V_f(a,x)$ *if* $a < x \le b$ and V(a) = 0. Prove that V and V - f are increasing functions on [a,b].
