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K19P 1517

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS-Reg./Suppl./Imp.) Examination, October - 2019 (2017 Admission Onwards) MATHEMATICS MAT 1C02 : LINEAR ALGEBRA

Time : 3 Hours

Max. Marks : 80

PART-A

Answer **Four** questions from this part. Each question carries **4** marks.

1. Let V be the real vector space of all functions f from $\mathbb{R}$ into $\mathbb{R}$. Check whether the set of all f such that f $(-1)=0$ is a subspace or not.
2. Let W_1 and W_2 be subspaces of a vector space V such that the set-theoretic unions of W_1 and W_2 is also a subspace. Prove that one of the spaces W_i is contained in the other.
3. Let V be the space of $nx1$ matrices over F and let W be the space of m×1 matrices over F. Let A be a fixed m×n matrix over F and let T be the linear transformation from V into W defined by $T(X)=AX$. Prove that T is the zero transformation if and only if A is the zero matrix.
4. Let T be the linear operator on $\mathbb{C}^2$ defined by $T(x_1,x_2)=(x_1,0)$. Let B be the standard ordered basis for $\mathbb{C}^2$ and let $B^1=\{\alpha_1,\alpha_2\}$ be the ordered basis defined by $\alpha_1=(1,i),\alpha_2=(-i,2)$, then what is the matrix of T in the ordered basis $\{\alpha_2,\alpha_1\}$?
5. Let V be a finite-dimensional vector space. What is the minimal polynomial for the identify operator on V?
6. Let V be an inner product space. The distance between two vectors α and β in V is defined by $d(\alpha,\beta)=||\alpha-\beta||$. Then show that $d(\alpha,\beta)\leq d(\alpha,\gamma)+d(\gamma,\beta)$.

P.T.O.

PART-B

Answer 4 questions from this part without omitting any unit. Each question carries **16** marks.

UNIT-I

7. a) If W is a subspace of a finite-dimensional vector space V, then prove that every linearly independent subset of W is finite and is part of a basis for W.

   b) Let A and B be mxn matrix over the field F. Then prove that A and B are row equivalent if and only if they have the same row space.

   c) Define rank of a linear transformation.

8. a) Define invertible linear transformation and give an example. Let V and W be vector space over the field F and let T be a linear transformation from V into W. If T is invertible, then prove that the inverse function is a linear transformation from W into V.

   b) Let V, W, and Z be vector space over the field F. Let T be a linear transformation from V into W and U be a linear transformation from W into Z. If B, B' and B'' are ordered bases for the spaces V, W and Z, respectively, if A is the matrix of T relative to the pair B, B' and B is the matrix of U relative to the pair B, B'', then prove that the matrix of the composition UT relative to the pair B, B'' is the product matrix $C=BA$.

9. a) Let V be a finite dimensional vector space over the field F, and let W be a subspace of V. Then show that dim W+dim W^0=dim V.

   b) If S is any subset of a finite-dimensional vector space V, then show that $(S^0)^0$ is the subspace spanned by S.

UNIT-II

10. a) Define characteristic space.

    b) Let T be a linear operator on a finite dimensional space V. Suppose that $T\alpha = c\alpha$. If f is any polynomial, then prove that $f(T)\alpha = f(c)\alpha$.

    c) Let T be a linear operator on a n-dimensional space V, and suppose that T has distinct characteristic values. Prove that T is diagonalizable.

11. a) Define minimal polynomial for a linear operator on a finite dimensional vector space.

b) Let T be a linear operator on a finite dimensional vector space V. If f is the characteristic polynomial for T, then prove that $f(T)=0$.

12. a) Let T is any linear operator on a vector space V and W be an invariant subspace for T. Then show that the characteristic polynomial for the restriction operator T_w divides the characteristic polynomial for T and the minimal polynomial for T_w divides the minimal polynomial for T.

b) Let V be a finite dimensional vector space over the field F. Let F be a commuting family of triangulable linear operator on V. Then prove that there exists an ordered basis for V such that every operator in F is represented by a triangular matrix in that basis.

UNIT-III

13. a) Define projection operator.

b) Let T be a linear operator on the space V, and let $W_1, ...W_k$ and $E_1,..., E_k$ satisfies.

i) Each E_i is a projection.

ii) $E_iE_j = 0 if i \neq j;$

iii) $I = E_1 + ... + E_k;$

iv) The range of $E_i is W_i$.

Then prove that a necessary and sufficient conditions that each subspace W_i be invariant under T is that T commutes with each of the projections Ei

c) If T is a linear operator on a finite dimensional vector space, then prove that every T-admissible subspace has a complementary subspace which is also in variant under T.

14. State and prove cyclic decomposition theorem.

15. a) State and prove Gram-Schmidt orthogonalization process.

b) Define orthogonal projection and give an example.
