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K19P 1520

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS - Reg./Suppl./Imp.) Examination, October - 2019
(2017 Admission onwards)
MATHEMATICS
MAT1C05:DIFFERENTIAL EQUATIONS

Time : 3 Hours   Max. Marks : 80

Instructions: Answer any **Four** questions from part A. Each question carries **4** marks. Answer any **Four** questions from part B without omitting any unit . Each question carries **16** marks.

PART-A

1. Find a power series solution in the form $\sum a_n x^n$ for the differential equation $y' = 2xy$. Verify your solution by solving the equation directly.

2. Define F(a,b,c,x) and show that $\sin^{-1} x = xF(\frac{1}{2},\frac{1}{2},\frac{3}{2},x^2)$.

3. State Rodrigues' formula for Legendre polynomial, use it to compute $P_0(x)$,$P_1(x)$ and $P_2(x)$.

4. Show that $x=e^{4t}$, $y=e^{4t}$ and $x=e^{-2t}$, $y=-e^{-2t}$ are solutions of the system $\frac{dx}{dt} = x + 3y, \frac{dy}{dt} = 3x + y$ and that these solutions are linearly independent on every closed interval.

5. Explain how to reduce the differential equation $y'' + P(x)y' + Q(x)y = 0$ to the normal form.

6. Starting with $y_0(x)=1$ apply Picard's method to find $y_1(x)$ and $y_2(x)$ for the initial value problem $y' = y^2$, y(0)=1.

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PART-B

UNIT-I

7. a) Let x_0 be an ordinary point of $y'' + P(x)y' + Q(x)y = 0$ and a_0 and a_1 are arbitrary constants. Prove that there exists a unique function analytic at 0, which is a solution of the differential equation in a neighborhood of 0 satisfying the initial conditions $y(0)=a_0$ and $y'(0) = a_1$.

   b) Find the general solution of $y'' + xy = 0$ about the ordinary point x=0.

8. a) Verify that origin is a regular singular point of the equation $4xy'' + 2y' + y = 0$. Also find two independent Frobenius series solutions.

   b) Find two independent Frobenius series solutions of $xy'' - y' + 4x^3 y = 0$.

9. a) Define hypergeometric series and derive this series as a solution of Gauss' hypergeometric equation.

   b) Verify that the Gauss' hypergeometric equation has $x = \infty$ as a regular singular point with exponents a and b.

UNIT-II

10. a) Derive the recursion formula for Legendre polynomials $(n+1)P_{n+1}(x)=(2n+1)x\,P_n(x)-nP_{n-1}(x)$.

    b) Establish the orthogonal property of Legendre polynomials

$$\int_{-1}^{1} P_m(x)P_n(x) = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1} & \text{if } m = n \end{cases}$$

    c) Find the first three terms of the Legendre series of $f(x)=e^x$.

11. a) Show that $\frac{d}{dx}[J_0(x)] = -J_1(x)$. Deduce that between any two positive zeros of $J_0(x)$ there is a zero of $J_1(x)$.

    b) Prove that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$ and $\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$. Using these derive the recurrence formula $\frac{2p}{x} J_p(x) = J_{p-1}(x) + J_{p+1}(x)$.

12. a) If the two solutions $x=x_1(t)$, $y=y_1(t)$ and $x=x_2(t)$, $y=y_2(t)$ of the system $\frac{dx}{dt} = a_1(t)x + b_1(t)y$, $\frac{dy}{dt} = a_2(t)x + b_2(t)y$ have a Wronskian that does not vanish on [a,b], then prove that $x=c_1x_1(t)+c_2x_2(t)$, $y=c_1y_1(t)+c_2y_2(t)$ is the general solution of the system on [a,b] for any constants c_1 and c_2.

b) Find the general solution of the system $\frac{dx}{dt} = 3x - 4y$, $\frac{dy}{dt} = x - y$.

UNIT-III

13. a) State and prove the sturm separation theorem.

b) Let u(x) be a nontrivial solution of $u'' + q(x)u = 0$, where $q(x)>0$ for all $x>0$. If $\int_1^\infty q(x)dx = \infty$, prove that u(x) has infinitely many zeros on the positive x-axis.

14. Let f(x,y) and $\frac{\partial t}{\partial y}$ be continuous functions of x and y on a closed rectangle R with sides parallel to the axes. If (x_0,y_0) is any interior point of R, prove that there is a number h with the property that the initial value problem $y' = f(x,y)$, $y(x_0)=y_0$ has a unique solution on the interval $|x - x_0| \leq h$.

15. a) Show that $f(x,y)=xy^2$.

i) Satisfies a Lipschitz condition on any rectangle $a \leq x \leq b$ and $c \leq y \leq d$.

ii) does not satisfy a Lipschitz condition on any strip $a \leq x \leq b$, $-\infty < y < \infty$.

b) Solve the system of first order equations by Picard's method.

$$\frac{dy}{dx} = z,\ y(0) = 1$$

$$\frac{dz}{dx} = -y,\ z(0) = 0.$$