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K19P 1519

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS-Reg./Sup./Imp.)
Examination, October - 2019
(2017 Admn. Onwards)
MATHEMATICS
MAT1C04 : BASIC TOPOLOGY

Time : 3 Hours Max. Marks : 80

Instructions:

Answer any **Four** questions from Part-A. Each question carries **4** marks
Answer any **Four** questions from Part-B without omitting any unit. Each question carries **16** marks

PART-A

1. Give an example of a set X and topologies T_1 and T_2 on X such that $T_1 \cup T_2$ is not a topology on **X.**
2. Consider $\mathbb{R}$ with the usual metric and $\mathbb{Q}$ with the subspace metric. Is, $\mathbb{Q}$ of the first category? Why?
3. Prove that the first countability axiom is a hereditary property.
4. Let υ be the usual topology on $\mathbb{R}$. Describe the weak topology on $\mathbb{R}$ induced by the function $i : \mathbb{R} \to (\mathbb{R}, \upsilon)$ defined by $i(x) = x$.
5. Prove that the closed interval [0, 1] has the fixed point property.
6. Prove that continuous image of a pathwise connected space is Path wise connected.

PART-B

Unit-I

7. a) Define a subbasis for a topology on a set X. Illustrate with an example.

P.T.O.

b) Let X be a set and ζ be a collection of subsets of X such that $X = \cup\{s : s \in \zeta\}$. Prove that there is a unique topology T on X such that ζ is a subbasis for T.

c) Define

i) First countable space

ii) Second countable space

iii) Prove that every second countable space is first countable. Is the converse true? Justify your answer

8. a) Let A be a subset of a topological space (X, T) and let $x \in X$. Prove that $x \in \bar{A}$ if and only if every neighborhood of x has a non empty intersection with A. Also prove that $\bar{A} = A \cup A'$.

b) Let A,B be subsets of a topological space (X,T) prove that $\mathrm{int}(A) \cup \mathrm{int}(B) \subseteq \mathrm{int}(A \cup B)$ and show by an example that equality need not hold.

c) Let $X = \{1,2,3,4,5\}$ and $T = \{\phi, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4,5\}, X\}$. Find the closed subsets of with respect to T.

9. a) Prove that every convergent sequence in a metric space is a Cauchy sequence.

b) Let (X,d) be a complete metric space and let A be subset of X with the subspace metric ρ. Prove that (A, ρ) is complete if and only if A is a closed subset of X.

c) Let (x,T) and (y, υ) be topological spaces. Define a continuous function $f : X \to Y$. If (X,T) is first countable and for each $x \in X$ and each sequence $\langle x_n \rangle$ in x such that $x_n \to x$, the sequence $\langle f(x_n) \rangle$ converges to $f(x)$, then prove that f is continuous.

Unit-II

10. a) Let (A,T_A) be a subspace of a topological space (X,T). Prove that a subset C of A is closed in (A,T_A) if and only if there is a closed subset D of (X,T) such that $C = A \cap D$.

b) Prove or disprove : Separability is a hereditary property.

c) Define an embedding one topological space in another topological space and show that $\mathbb{R}$ with usual topology can be embedded in $\mathbb{R}^2$ with the usual topology

11. a) Define the product space of two topological spaces (X,T_1) and (Y,T_2). Let (X_1,d_1) and (X_2,d_2) be metric spaces, for each i = 1,2, Let T_i be the topology on X_i generated by d_i. Prove that the product topology on $X = X_1 \times X_2$ is same as the topology on X generated by the product metric.

b) Let $(X,T),(Y_1,\upsilon_1)$ and (Y_2,υ_2) be topological spaces and let $f_1 : X \to Y_1$ and $f_2 : X \to Y_2$ be functions , and define $f : X \to Y_1 \times Y_2$ by $f(x) = (f_1(x), f_2(x))$. Prove that f is continuous if and only if f_1 and f_2 are continuous.

12. a) Let $\{(X_\alpha,T_\alpha):\alpha \in \wedge\}^2$ be an indexed family of topological spaces and for each α in $\wedge$ let (A_α,T_{A_α}) be a subspace of (X_α,T_α). Prove that the product topology on $\prod_{\alpha\in\wedge} A_\alpha$ is same as the subspace topology on $\prod_{\alpha\in\wedge} A_\alpha$ determined by the product topology on $\prod_{\alpha\in\wedge} X_\alpha$.

b) Let $\{(X_\alpha,T_\alpha):\alpha \in \wedge\}$ be a family of topological spaces and let $X = \prod_{\alpha\in\wedge} X_\alpha$. Prove that the product space (X,T) is second countable if and only if (X_α,T_α) is second countable for all $\alpha \in \wedge$ and T_α is the trivial topology for all but a countable number of α.

Unit-III

13. a) Let (X,T) be a topological space and let $A \subseteq X$. Prove that the following conditions are equivalent.

i) The subspace (A,T_A) is connected.

ii) The set A cannot be expressed as the union of two non empty sets that are separated in X.

iii) There do not exist $U,V \in T$ such that $U \cap A \neq \phi$, $V \cap A = \phi$, $U \cap V \cap A = \phi$ and $A \subset U \cup V$.

b) Let T be the usual topology on $\mathbb{R}$. Prove that $(\mathbb{R},T)$ is connected.

14. a) Define a simple chain in a topological space (X,T) and a covering of X. Let (X,T) be a connected space, let O be an open cover of X and let a,b be distinct points of X. Prove that there is a simple chain consisting of members of O that connects a and b.

b) Define a pathwise connected space and show that the topologist's sine curve is not pathwise connected.

15. a) Define a locally connected space. Prove that a topological space is locally connected if and only if each component of each open set is open.

b) When is a topological space said to be

i) Totally disconnected

ii) 0-dimensional

iii) a T_0- Space?

c) Prove that every 0-dimensional T_{∞}- Space is totally disconnected.