



0043013

K19P 1516

Reg. No. :

Name :

I Semester M.Sc. Degree (CBSS-Reg./Suppl./Imp.)

Examination, October - 2019

(2017 Admission Onwards)

MATHEMATICS

MAT1C01: BASIC ABSTRACT ALGEBRA

Time : 3 Hours

Max. Marks : 80

PART-AAnswer any **Four** questions from this part. Each question carries **4** marks.

1. Prove that every ideal of \mathbb{Z} is principal.
2. Give addition and multiplication tables of the ring $2\mathbb{Z} / 8\mathbb{Z}$
3. Find all abelian groups up to of order 16.
4. Find the ascending central series of D_4 .
5. Let X be a G -set. Then prove that $G_X = \{g \in G \mid gx = x\}$ is a subgroup of G .
6. Find the Sylow 3-subgroups of \mathbb{Z}_{12} .

PART - BAnswer **Four** questions from this part without omitting any unit. Each question carries **16** marks.**UNIT- I**

7. a) Define decomposable group. Prove that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime.
b) State and prove Cauchy's Theorem.
8. a) Let X be a G -set. For each $g \in G$, prove that the function $\sigma_g : X \rightarrow X$ defined by $\sigma_g(x) = gx$ is a permutation of X . Hence prove that the map $\phi : G \rightarrow S_X$ defined by $\phi(g) = \sigma_g$ is a homomorphism.
b) For a prime p , prove that every group G of order p^2 is abelian.

P.T.O.



9. a) Let G be a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $H \vee K = G$. Then prove that G is isomorphic to $H \times K$.
- b) Prove that no group of order 48 is simple.

UNIT-II

10. a) Let F be the field of quotient of an integral domain D . Then prove that the map $i: D \rightarrow F$ defined by $i(a) = [(a, 1)]$ is an isomorphism of D with a subring of F .
- b) State and prove second isomorphism theorem in group theory.
11. a) Let K and L be normal subgroup of G with $K \vee L = G$, and $K \cap L = \{e\}$. Show that $G/K \cong L$ and $G/L \cong K$.
- b) If G has a composition series and N is a proper normal subgroup of G , then prove that there exists a composition series containing N . Give an example of a composition series.
12. a) Let G be a nonzero free abelian group with a finite basis. Then prove that every basis of G is finite and all basis have same number of elements.
- b) Show that a free abelian group contains no nonzero elements of finite order.

UNIT-III

13. a) If G be a finite subgroup of the multiplicative group $\langle F^* \cdot \rangle$ of a field F . Then prove that G is cyclic.
- b) State and prove Eisenstein Criterion.
14. a) The polynomial X^4+4 can be factored into linear factors in $\mathbb{Z}_5[x]$. Find this factorization.
- b) State and Prove the evaluation homomorphism theorem for field theory.
15. a) Let R be a commutative ring with unity. Then prove that if M is a maximal ideal of R if and only if R/M is a field.
- b) If F is a field, prove that every non trivial prime ideal in $F[x]$ is principal.
-