K18P 1431

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (Reg./Suppl./Imp.) Examination, October 2018
(2017 Admn. Onwards)
MATHEMATICS
MAT1C03 : Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Prove that set of all sequences whose elements are digits 0 and 1 is uncountable.

2. Discuss the continuity of the function $f(x) = \begin{cases} x & \text{if } x \text{ is irrational} \\ 0 & \text{if } x \text{ is rational} \end{cases}$.

3. Evaluate $\lim\limits_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

4. If $f \in R(\alpha)$ on [a, b], prove that $|f| \in R(\alpha)$ on [a, b].

5. Let $f \in R$ on [a, b] and for $a \le x \le b$, let $F(x) = \int_a^x f(t)\,dt$. Prove that F is continuous on [a, b].

6. Examine whether the function given by $f(x) = \begin{cases} \sqrt{x}\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is of bounded variation on [0, 1].

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

UNIT – I

7. a) Define convex set. Prove that closed balls in $\mathbb{R}^k$ are convex.

   b) Prove that compact subsets of metric space are closed.

P.T.O.

8. a) Define perfect set. Show that Cantor set is perfect.

b) Give an example of continuous and unbounded function on (0, 1) and continuous and bounded on (0, 1).

9. a) Let $X = [0, 2\pi)$ and Y is a unit circle centred at the origin. Let $f : X \to Y$ be defined by $f(t) = (\cos t, \sin t)$. Is f continuous ? Does f^{-1} exist ? If it exists, is it continuous ? Justify your answer.

b) Let f be monotonic on (a, b). Show that the set of all points of (a, b) at which f is discontinuous is atmost countable.

UNIT – II

10. a) Let f be continuous on [a, b], $f'(x)$ exists at some point $x \in [a, b]$, g is defined on the interval I which contains the range of f and g is differentiable at the point f(x). If $h(t) = g(f(t))$, $a \leq t \leq b$. Prove that h is differentiable at x. Prove that h is differentiable at x and $h'(x) = g'(f(x))f'(x)$.

b) Check the continuity of f(x), if $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

11. a) State and prove generalised mean value theorem.

b) If $f : [0, 1] \to \mathbb{R}$ defined by $f(x) = \begin{cases} 1 & \text{if x is rational} \\ 0 & \text{if x is irrational} \end{cases}$ find $\underline{\int_a^b} f dx$ and $\overline{\int_a^b} f dx$.

12. a) Suppose $f \in R(\alpha)$ on [a, b], $m \leq f \leq M$, ϕ is continuous on [m, M] and $h(x) = \phi(f(x))$ on [a, b]. Prove that $h \in R(\alpha)$ on [a, b].

b) Suppose α is increases monotonically $\alpha' \in R$ on [a, b] and f is bounded real function on [a, b]. Show that $f \in R(\alpha)$ on [a, b] if and only if $f\alpha' \in R$.

UNIT – III

13. a) Let $f : [a, b] \to \mathbb{R}^k$ and if $f \in R(\alpha)$ for some monotonically increasing function α on [a, b], prove that $|f| \in R(\alpha)$ on [a, b] and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| \, d\alpha$.

    b) State and prove fundamental theorem of integral calculus.

14. a) Let f be monotonic on [a, b]. Show that the set of discontinuities of f is countable.

    b) Let f be of bounded variation on [a, b] and $c \in$ (a.b). Prove that f is of bounded variation on [a, c], on [c, b] and $V_f(a.b) = V_f(a.c) + V_f(c.b)$.

15. a) If f is continuous on [a, b] and f′ exist and is bounded in (a, b). Prove that f is of bounded variation on [a, b].

    b) Find the length of the curve $f(t) = e^{2\pi i t}$, $t \in [0, 2]$.
