K18P 1433

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (Reg./Suppl./Imp.) Examination, October 2018
(2017 Admn. Onwards)
MATHEMATICS
MAT1C 05 : Differential Equations

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any four** questions. **Each** question carries **4** marks.

1. Find a power series solution of the form $\sum a_n x^n$ for the differential equation $y' = y$.
2. Find the indicial equation and its roots for the differential equation $x^3 y'' + (\cos 2x - 1)y' + 2xy = 0$.
3. Derive the recursion formula $(n + 1) P_{n+1}(x) = (2n + 1) x P_n(x) - n P_{n-1}(x)$ for Legendre polynomials.
4. For an integer $m \geq 0$, prove that $J_{-m}(x) = (-1)^m J_m(x)$.
5. Find the normal form of the Bessel equation $x^2 y'' + xy' + (x^2 - p^2)y = 0$.
6. Starting with $y_0(x) = 1$ use Picard's method to calculate $y_1(x)$, $y_2(x)$ and $y_3(x)$ for the problem $y' = y^2$, $y(0) = 1$. **(4×4=16)**

PART – B

Answer **any four** questions without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Find the general solution of the Airy's equation $y'' + xy = 0$ as a power series $y = \sum a_n x^n$.

   b) Verify that origin is a regular singular point of the equation $2x^2 y'' + x(2x + 1) y' - y = 0$ and calculate two independent Frobenius series solution.

P.T.O.

8. a) Show that the equation $x^2y'' - 3xy' + (4x+4)y = 0$ has only one Frobenius solution. Find it.

   b) Find the general solution of $(1+x^2)\,y'' + 2xy' - 2y = 0$ in terms of power series in x. Can you express this solution by means of elementary functions ?

9. a) Define Gauss's hypergeometric equation and obtain the hypergeometric series as a solution of this equation.

   b) Show that (i) $\sin^{-1}x = x\,F\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, x^2\right)$ (ii) $e^x = \lim_{b\to\infty} F\left(a, b, a, \frac{x}{b}\right)$.

Unit – II

10. a) Derive Rodrigue's formula for Legendre polynomials. Use the formula to write the first four Legendre polynomials.

    b) If $P_m(x)$ and $P_n(x)$ are Legendre polynomials, prove that

$$\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$$

11. a) Prove that :

    i) $\frac{d}{dx}\left(x^P J_P(x)\right) = x^P J_{P-1}(x)$

    ii) $\frac{d}{dx}\left(x^{-P} J_P(x)\right) = -x^{-P} J_{P+1}(x)$

    iii) $2J_P'(x) = J_{P-1}(x) - J_{P+1}(x)$

    b) With usual notation prove that

$$\int_0^1 x\,J_P(\lambda_m(x))J_P(\lambda_n(x))\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{1}{2}J_{P+1}(\lambda_n)^2 & \text{if } m = n \end{cases}$$

12. a) If $x = x_1(t)$, $y = y_1(t)$ and $x = x_2(t)$, $y = y_2(t)$ are two linearly independent solutions on [a, b] of the system $\frac{dx}{dt} = a_1x + b_1y$, $\frac{dy}{dt} = a_2x + b_2y$, then prove that $x = c_1x_1(t) + c_2x_2(t)$, $y = c_1y_1(t) + c_2y_2(t)$ is the general solution on [a, b] for any constants c_1 and c_2.

    b) Find the general solution of the system $\frac{dx}{dt} = -3x + 4y$, $\frac{dy}{dt} = -2x + 3y$.

Unit – III

13. a) State and prove sturm separation theorem.

b) Let y(x) and z(x) be nontrivial solutions of $y'' + q(x)\, y = 0$ and $z'' + r(x)\, z = 0$, where q(x) and r(x) are positive functions such that q(x) > r(x). Prove that y(x) vanishes at least once between any two successive zeros of z(x).

14. a) Find the exact solution of the initial value problem $y' = 2x\,(1 + y)$, $y(0) = 0$. Starting with $y_0(x) = 0$, calculate $y_1(x)$, $y_2(x)$, $y_3(x)$ and compare these results with the exact solution.

b) Solve the following system using Picard's method and compare the result with the exact solution.

$$\frac{dx}{dt} = z,\ y(0) = 1;\ \frac{dz}{dx} = -y,\ z(0) = 0.$$

15. a) Let f(x, y) be a continuous function that satisfies a Lipschitz condition $|f(x, y_1) - f(x, y_2)| \le K\, |y_1 - y_2|$ on a strip $a \le x \le b$, $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, prove that the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ has a unique solution on the interval $a \le x \le b$.

b). Show that $f(x, y) = xy^2$ satisfies Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$. **(4×16=64)**
