K18P 1429

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (Reg./Suppl./Imp.) Examination, October 2018
(2017 Admn. Onwards)
MATHEMATICS
MAT 1C01 : Basic Abstract Algebra

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **four** questions from this part. **Each** question carries **4** marks.

1. Prove that direct product of abelian groups is abelian.
2. Prove that a group of order 16 is not simple.
3. Find the product of the polynomials $f(x) = 4x - 5$ and $g(x) = 2x^2 - 4x + 2$ in $\mathbb{Z}_8[x]$.
4. Let $\phi : \mathbb{Z}_{18} \to \mathbb{Z}_{12}$ be the homomorphism where $\phi(1) = 10$. Find the kernel of ϕ.
5. Define prime ideal. Give an example of a prime ideal which is not maximal.
6. Give an example to show that a factor ring of an integral domain may have divisors of zero.

PART – B

Answer **four** questions from this part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that if the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.
   b) Let X be a G-set and let $x \in X$. Then prove that $|Gx| = (G : G_x)$.
8. a) Let $G = S_3$ be the group of all permutations of the set $X = \{1, 2, 3\}$.
      i) Then prove that X is a G-set with respect to the operation $\sigma.x = \sigma(x)$.
      ii) Find the subgroup G_1.
      iii) Find X_ρ where $\rho = (1,2)$.
   b) State and prove Cauchy's theorem.

P.T.O.

9. a) Prove that every group of prime power order is solvable.

   b) Prove that no group of order 36 is simple.

Unit – II

10. a) Let D is an integral domain and $S = \{(a, b) | a, b \in D, b \neq 0\}$. Then show that the relation ~ defined by (a, b) ~ (c, d) if and only if ad = bc is an equivalence relation.

    b) If N is a normal subgroup of G and if H is any subgroup of G, then prove that $H \vee N = HN = NH$. Further prove that, if H is normal subgroup of G, then HN is a normal subgroup of G.

11. a) Define isomorphic normal series of a group G. Given an example.

    b) Let H and K be subgroups of a group G and let H^* and K^* be normal subgroup of H and K, respectively. TheN prove that $H^*(H \cap K^*)$ is a normal subgroup of $H^*(H \cap K)$.

12. a) If G has a composition series and N is a proper normal subgroup of G, then prove that G has composition series containing N.

    b) Prove that every finitely generated abelian group is isomorphic to a group of the form $\mathbb{Z}_{m_1} \times \mathbb{Z}_{m_2} \times \ldots \times \mathbb{Z}_{m_r} \times \mathbb{Z} \times \ldots \times \mathbb{Z}$ where m_i divides m_{i+1} for $i = 1, \ldots, r-1$.

Unit – III

13. a) State and prove division algorithm for a polynomial ring F[x] over a filed F.

    b) Show that for p a prime, the polynomial $x^p + a$ in $\mathbb{Z}_p[x]$ is irreducible for any $a \in \mathbb{Z}_p$.

14. a) Find all ideals N of $\mathbb{Z}_{12}$. In each case compute $\mathbb{Z}_{12}/N$.

    b) Let $\phi : R \to R'$ be a ring homomorphism and let N be an ideal of R and N' be an ideal of R'.

       i) Prove that $\phi(N)$ is an ideal of $\phi(R)$.

       ii) Prove that $\phi^{-1}(N')$ is an ideal of R.

15. a) Prove that every maximal ideal in commutative ring with unity is a prime ideal. What about the converse ? Justify your answer.

    b) If F is a field, prove that every ideal in F[x] is principal ideal.
