K21U 6804

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2021
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01 MAT-BCA : Mathematics for BCA I

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark.

1. Derive the derivative of tan x.

2. Find the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

3. Write the dual of the following statement.

   $a + a'b = a + b$.

4. If the rank of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & \lambda \end{bmatrix}$ is 1, find λ.

5. If A is an orthogonal square matrix, then prove that $|A| = \pm 1$.

PART – B

Answer **any 7** questions from this Part. **Each** question carries **2** marks.

6. Find the derivative of $\sqrt{\sin\sqrt{x}}$.

7. If $y = \sin^{-1} x$, prove that $(1 - x^2)\, y_2 - 2xy_1 = 0$.

8. Find the n^{th} derivative of $e^{2x} \sin x \sin 2x$.

P.T.O.

9. If $x = \frac{1}{2}\left(t + \frac{1}{t}\right), y = \frac{1}{2}\left(t - \frac{1}{t}\right)$, find $\frac{d^2y}{dx^2}$.

10. Prove that in a Boolean algebra B, a + 1 = 1 for all $a \in B$.

11. Show that the power set of A = {a, b} is a Boolean algebra.

12. Solve the system of equations $x + y + z = 3$, $2x + 4y - z = 0$, $x - 3y + 2z = 5$.

13. Find value of a and b, if $A = \frac{1}{\sqrt{2}}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ is orthogonal.

14. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 1 & 1 & 4 \end{bmatrix}$.

15. Test for consistency the equations $x + y + z = 2$, $x + 2y + 3z = 4$, $x + 3y + 4z = 5$.

PART – C

Answer **any 4** questions from this Part. **Each** question carries **3** marks.

16. Derive the derivative of $\cos^{-1} x$.

17. Find $\frac{dy}{dx}$, if $y = \frac{x^{\frac{1}{2}}(1-2x)^{\frac{2}{3}}}{(2-3x)^{\frac{3}{4}}(3-4x)^{\frac{4}{5}}}$.

18. If $x^3 + y^3 = 3axy$, prove that $\frac{d^2y}{dx^2} = -\frac{2a^2xy}{(y^2 - ax)^3}$.

19. Find the n^{th} derivative of $\frac{1}{x^2 + a^2}$ in terms of r and θ.

20. State and prove absorption laws.

21. Find the value of λ and μ so that the system of equations $4x + 5y + 6z = 16$, $x - 5z = -9$, $x + 2y + \lambda z = \mu$ has (i) no solution, (ii) unique solution, (iii) infinite number of solutions.

22. Are the vectors $x_1 = (1, 3, 4, 2)$, $x_2 = (3, -5, 2, 2)$, $x_3 = (2, -1, 3, 2)$, linearly independent ? If so, express one of these as a linear combination of the others.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **5** marks.

23. Find the derivatives of the following.

a) $y = \frac{x \sin^{-1} x}{\sqrt{1-x^2}}$.

b) $x^{\tan x} + (\sin x)^{\cos x}$.

24. If $y = e^{a \cos^{-1} x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (a^2 + n^2)y_n = 0$. Further, find $(y_n)_0$.

25. Show that the following statements are equivalent in a Boolean algebra.

a) $a + b = a$

b) $a * b = b$

c) $a + b = 1$

d) $a * b' = 0$.

26. a) Using Gauss-Jordan method find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

b) Solve by Cramer's rule the system of equations $4x + 5y + 6z = 16$, $x - 5z = -9$, $x + 2y + 3z = 7$.
