K22U 0129

Reg. No. : ...............................

Name : ...................................



**VI Semester B.Sc. Degree (CBCSS – Supple./Improv.) Examination, April 2022**
**(2016 – 2018 Admissions)**
**CORE COURSE IN MATHEMATICS**
**6B12MAT : Complex Analysis**

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. Write $\frac{1}{2-5i}$ in the form x + iy.

2. Write $e^z$ in the form u + iv, if $z = 2 + 3\pi i$.

3. Define radius of convergence of a power series.

4. Give an example of a function having double zero at z = 1.

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. Show that $u = -e^{-x} \sin y$ is harmonic.

6. Express $-3 + 3i$ in the exponential form.

7. Find the value of $\ln(2 - i)$.

8. Find a parametric representation $z = z(t)$ for the upper half of $|z - 4 + 2i| = 3$.

P.T.O.

9. Integrate $\frac{z^3}{2z - i}$ counter clockwise around the unit circle.

10. Write Cauchy's inequality.

11. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z - 3i)^n$.

12. Is the series $\sum_{n=0}^{\infty} \frac{(100 + 75i)^n}{n!}$ convergent ? Justify your answer.

13. State Laurent's theorem.

14. Find $\text{Res}_{z=0} \frac{\sin 2z}{z^4}$.

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

15. Find all roots of $\sqrt[3]{1+i}$.

16. Evaluate $\int_C \bar{z}\, dz$ where C is the arc from 0 to 1 + i along the parabola $y = x^2$.

17. Evaluate $\int_C \frac{2z^3 + z^2 + 4}{z^4 + 4z^2} dz$ where C is the circle $|z - 2| = 4$, clockwise.

18. State and prove Morera's Theorem.

19. Find all Laurent series of $\frac{1}{z^3 - z^4}$ with center zero.

20. Show that the zeros of an analytic function are isolated.

## SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

21. Verify that $u = x^2 - y^2 - y$ is harmonic and find its harmonic conjugate.

22. If f(z) is analytic in a simply connected domain D, then show that there exist a function F(z) such that $F'(z) = f(z)$ which is analytic in D. Also prove that $\int_C f(z)dz = F(z_1) - F(z_0)$ where C is any path from $z_0$ to $z_1$ in D.

23. a) Prove that a power series with non zero radius of convergence is the Taylor series of its sum.

    b) Find a Taylor series expansion about i of $\frac{1}{z}$.

24. State Residue theorem. Using that evaluate $\int_C \left( \frac{ze^{\pi z}}{z^4 - 16} + ze^{\frac{\pi}{z}} \right) dz$ where C is the ellipse $9x^2 + y^2 = 9$ (counter clockwise).