



K21U 1125

Reg. No. :

Name :



IV Semester B.Sc. Degree CBCSS (Core) Regular Examination, April 2021
(2019 Admission Only)
CORE COURSE IN MATHEMATICS
4B04 MAT : Number Theory and Applications of Integrals

Time : 3 Hours

Max. Marks : 48

PART – A
(Short Answer Type)

Answer **any four** out of five questions. **Each** question carries 1 mark.

1. State the Division Algorithm.
2. Find the lcm (31, 25).
3. Check whether the Diophantine equation $6x + 51y = 22$ is solvable.
4. What is a pseudoprime ? Give an example.
5. State the Euler's theorem. (4×1=4)

PART– B
(Short Essay Type)

Answer **any eight** out of eleven questions. **Each** question carries 2 marks.

6. If $a|bc$, with $\gcd(a, b) = 1$, show that $a|c$.
7. Find the gcd (12378, 3054).
8. For any choice of positive integers a and b , show that $\text{lcm}(a, b) = ab$ if and only if $\gcd(a, b) = 1$.
9. Give an example to show that $a^2 \equiv b^2 \pmod{n}$ need not imply that $a \equiv b \pmod{n}$.
10. If p is a prime, then-show that $a^p \equiv a \pmod{p}$ for any integer a .

P.T.O.



11. Evaluate $\int_0^{\pi/4} \tan x \sec^2 x \, dx$.
12. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.
13. Find the area of the region in the xy – plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.
14. The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$, and the x -axis is revolved about the x -axis to generate a solid. Find its volume.
15. Find the volume of the solid generated by revolving the region between the y -axis and the curve $x = \frac{2}{y}$, $1 \leq y \leq 4$, about the y -axis.
16. The circle $x^2 + y^2 = a^2$ is rotated about the x -axis to generate a sphere. Find its volume. (8×2=16)

PART – C
(Essay Type)

Answer **any four** out of seven questions. **Each** question carries **4** marks.

17. Determine all solutions in the integers of the Diophantine equation $5x + 22y = 18$.
18. For any positive integers a and b , show that $a \equiv b \pmod{n}$ if and only if a and b leave the same remainder when divided by n .
19. If p is a prime, prove that for any integer a ,
 $p \mid a^p + a(p-1)!$ and
 $p \mid a^p(p-1)! + a$.
20. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
21. Find the length of the graph of $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \leq x \leq 4$.
22. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x – axis to generate a solid. Find the volume of the solid.
23. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$, about the x – axis. (4×4=16)



PART – D
(Long Essay Type)

Answer **any two** out of four questions. **Each** question carries **6** marks.

24. State and prove the Fundamental Theorem of Arithmetic.
 25. Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod{4}$.
 26. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x – axis and the line $y = x - 2$.
 27. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ about the line $x = 3$. **(2×6=12)**
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