K23U 2367

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2023
(2019-2021 Admissions)
CORE COURSE IN MATHEMATICS
5B07 MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark : **(4×1=4)**

1. Give an example of a finite group that is not cyclic.
2. Find the order of the element 4 in Z_6.
3. What is the order of the permutation (124) (23) in S_6 ?
4. Define Kernel of a homomorphism.
5. Find all solutions of the equation $x^2 + 2x + 2 = 0$ in Z_6.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks : **(8×2=16)**

6. Find the group table of the Klein 4-group. List all its subgroups.
7. Show that every cyclic group is abelian. Discuss its converse.
8. Let S be the set of all real numbers except – 1. Define $*$ on S by $a * b = a + b + ab$. Check whether $(S, *)$ is a group or not.
9. Find all the generators of Z_{18}.

P.T.O.

10. Find the number of elements in the set $\{\sigma \in S_5 | \sigma(2) = 5\}$.

11. Define odd permutation. Give an example of an odd permutation in S_4.

12. Prove that a group homomorphism ϕ defined on G is one-to-one if and only if $\ker(\phi) = \{e\}$.

13. Consider $\gamma : Z \rightarrow Z_n$ by $\gamma(m) = r$, where r is the remainder when m divided by n. Show that γ is a group homomorphism. What is its kernel ?

14. Show that the cancellation law with respect to multiplication hold in a ring R if and only if R has no divisors of zero.

15. Show that every field is an integral domain. Discuss its converse.

16. Define characteristic of a ring. What is the characteristic of the ring Z_6 ?

PART – C

Answer **any 4** questions from this Part. **Each** question carries 4 marks : (4×4=16)

17. Let G be a group and let a be one fixed element of G. Show that the set $H_a = \{x \in G | xa = ax\}$ is a subgroup of G.

18. Show that every permutation of a finite set can be written as a product of disjoint cycles.

19. Let G be a group of order pq, where p and q are prime numbers. Show that every proper subgroup of Z_{pq} is cyclic.

20. Let H be a subgroup of a group G such that $ghg^{-1} \in H$ for all $g \in G$ and all $h \in H$. Show that $gH = Hg$.

21. Let $\phi : G \rightarrow G'$ be a group homomorphism with kernel H and let $a \in G$. Show that $\{x \in G | \phi(x) = \phi(a)\} = aH$.

22. Show that the map $\phi : Z \rightarrow Z_n$ where $\phi(a)$ is the remainder of a modulo n is a ring homomorphism.

23. An element a of a ring R is idempotent of $a^2 = a$. Show that a division ring contains exactly two idempotent elements.

PART – D

Answer **any 2** questions from this Part. **Each** question carries 6 marks : (2×6=12)

24. State and prove Cayley's theorem.

25. Let H be a subgroup of a group G. Then show that the left coset multiplication (aH) (bH) = abH is well-defined if and only if H is a normal subgroup of G.

26. Show that if a finite group G has exactly one subgroup H of a given order, then H is a normal subgroup of G.

27. Show that the characteristic of an integral domain must be 0 or a prime number. Give examples of two non-isomorphic rings with characteristic 4.