



K21P 0557

Reg. No. :

Name :



First Semester M.Sc. Degree (CBSS-Reg./Suppl. (Including Mercy
Chance) /Imp.) Examination, October 2020
(2017 Admission Onwards)

MATHEMATICS

MAT 1C01 – Basic Abstract Algebra

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries 4 marks.

1. Define normal series. Give an example.
2. Show that every group of order 45 has a normal subgroup of order 9.
3. If R is a ring with unity 1, then prove that the map $\phi : \mathbb{Z} \rightarrow R$ given by $\phi(n) = n \cdot 1$ is a homomorphism.
4. Find the sum and the product of the polynomials $f(x) = 2x^2 - 4x + 2$ and $g(x) = 4x - 5$ in $\mathbb{Z}_5[x]$.
5. Write all abelian groups up to isomorphism of order 24.
6. Define isomorphic normal series. Give an example.

PART – B

Answer **any four** questions from Part – B. Without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to \mathbb{Z}_{mn} if and only if m and n are relatively prime.
b) Let G be an abelian group. Let H be the subset consisting of the identity e together with all elements of G of order 2. Show that H is a subgroup of G .

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8. a) Let X be a G -set. Then prove that $|Gx| = (G : G_x)$
 b) Show that there are no simple groups of order 255.
9. a) If H and K are subgroups of a group G , then prove that $|HK| = \frac{(|H|)(|K|)}{|H \cap K|}$.
 b) Show that no group of order 30 is simple.

Unit – II

10. a) Prove that any two fields of quotients of an integral domain D are isomorphic.
 b) Describe the field of quotients of the integral subdomain $D = \{n + mi \mid n, m \in \mathbb{Z}\}$ of \mathbb{C} .
11. a) If N is a normal subgroup of G and H is any subgroup of G , then prove that $H \vee N = HN = NH$. Also, if H is normal subgroup of G , then prove that HN is a normal subgroup of G .
 b) Show that the center of a group G is a normal subgroup of G . Find the ascending central series of D_4 .
12. a) Prove that every finitely generated abelian group is isomorphic to a group of the form $\mathbb{Z}_{m_1} \times \mathbb{Z}_{m_2} \times \dots \times \mathbb{Z}_{m_r} \times \mathbb{Z} \times \dots \times \mathbb{Z}$ where m_i divides m_{i+1} for $i = 1, \dots, r - 1$.
 b) Show that a free abelian group contains no nonzero elements of finite order.

Unit – III

13. a) State and prove division algorithm for $F[x]$.
 b) Show that the polynomial $f(x) = x^4 - 2x^2 + 8x + 1$ is irreducible over \mathbb{Q} .
14. a) Let R be a commutative ring with unity. Then prove that if M is a maximal ideal of R if and only if R/M is a field.
 b) Show that a factor ring of a field is either the trivial ring of one element or is isomorphic to the field.
15. a) Is $x^3 + 2x + 3$ an irreducible polynomial of $\mathbb{Z}_5[x]$? Why? Express it as product of irreducible factors.
 b) Prove that an ideal $\langle p(x) \rangle \neq \{0\}$ of $F[x]$ is maximal if and only if $p(x)$ is irreducible.