K20U 1535

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2020
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B08 MAT – Vector Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **four** questions are **compulsory**. **Each** question carries **1** mark :

1. Find the gradient of the function $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$ at (1, 1).

2. If $w = \sin(x + 2z)$ and $x^3 + z^3 = 3$, find $\frac{dw}{dx}$ using chain rule.

3. Evaluate $\int_C 9x^2y\, dx$ where C is given by $x = t^2$, $y = t^3$, $0 \le t \le 2$.

4. Write the formula for finding the surface area of a surface S given by $F(x, y, z) = C$, defined over the planar region R. **(4×1=4)**

SECTION – B

Answer **any 8** questions from questions **5** to **14**. **Each** question carries **2** marks :

5. Write the vector equation of a line in a plane passing through (1, 2) and making an angle $\frac{\pi}{3}$ with the positive X-axis.

6. Find the length of the curve $r(t) = 3\cos t\, i - 3\sin t\, j - 4t\, k$, $1 \le t \le 3$.

7. Find the directional derivative of $f(x, y) = e^{2xy}$ at (–2, 0) in the direction of i + j.

P.T.O.

8. If $w = x^2 + y^2 + z^2$ and $z^3 + xyz + yz^2 = 0$, find $\frac{\partial w}{\partial x}$ at (x, y, z) = (1, 1, 0) treating x and y as independent variables.

9. Find the linear approximation L(x, y) of the function $f(x, y) = e^{2y+x}$ at (2, 3).

10. If $\phi(x, y) = 3\sqrt{x^2 + y^2}$, find div(grad($\phi$)).

11. Find the flux of $F = (x - y)i + xj$ across the circle $x^2 + y^2 = 1$ in the XY-plane.

12. If the force F = 4xi + 4yj is acting on a particle moving it along the curve r(t) = ti + (1 + 2t)j from (1, 3) to (3, 7), find the work done by the force.

13. Find a parametrization of the surface of the paraboloid $z = 16 - x^2 - y^2$, $z \geq 0$.

14. State the Gauss divergence theorem. **(8×2=16)**

SECTION – C

Answer **any 4** questions from questions **15** to **20**. **Each** question carries **4** marks :

15. Find the equation of the plane through the points A(1, 0, 2), B(1, 1, 1), C(1, 2, 3).

16. Show that $\frac{d}{dt}(U.V) = \frac{d}{dt}(U).V + U.\frac{d}{dt}(V)$, where U, V are functions of t into $\mathbb{R}^2$.

17. Use the Taylor's formula for $f(x, y) = e^x \cos y$ at the origin to find the quadratic approximation of f. Hence find approximate value of f(0.1, 0.2).

18. Find Curl (F × G) at (1, 2, 0) where $F(x, y, z) = 3x^2 i + 2xyj + 2yzk$ and $G(x, y, z) = 4yzi + y^2 j + xyz\,k$.

19. Using Green's theorem, find the area enclosed by the circle $x^2 + y^2 = 4$.

20. Evaluate the surface integral $\iint_\sigma y^2 z^2 \, dS$, where σ is the part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the planes z = 1 and z = 2. **(4×4=16)**

SECTION – D

Answer **any 2** questions from questions **21** to **24**. **Each** question carries **6** marks :

21. Find the unit tangent vector, unit normal vector and the binormal vector at $t = 0$ for the curve $r(t) = 2\cos t\, i + 2\sin t\, j + 4tk$.

22. Find the absolute maximum and minimum values of the function $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices (0, 0), (3, 0) and (0, 5).

23. Check whether the vector field $F = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is conservative or not. If conservative, find its scalar potential function.

24. Verify Stoke's theorem for the vector field $F(x, y, z) = 2zi + 3xj + 5yk$, taking the surface to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for $z \geq 0$, with upward orientation, and the curve to be the positively oriented circle of intersection of the paraboloid with the XY-plane. **(2×6=12)**
