K23P 0501

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT2C09 : Foundations of Complex Analysis

Time : 3 Hours | Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries 4 marks.

1. Evaluate the integral $\int_{\gamma} \frac{dz}{z^2+1}$, where $\gamma(t) = 2e^{it}$, $0 \le t \le 2\pi$.

2. Define index of a closed rectifiable curve with respect to a point. Illustrate with example.

3. Determine the nature of the singularity of the function $f(z) = \frac{\log(z+1)}{z^2}$.

4. Find the residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$.

5. Define pole and essential singularity, giving one example of each.

6. State Arzela-Ascoli theorem in space of continuous functions.

PART – B

Answer **any 4** questions without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) If G is a region and $f : G \to \mathbb{C}$ is an analytic function such that there is a point a in G with $|f(a)| \ge |f(z)|$ for all z in G. Prove that f is constant.

   b) If G is simply connected and $f : G \to \mathbb{C}$ is analytic in G. Prove that f has primitive in G.

P.T.O.

8. a) Let γ be a closed rectifiable curve in $\mathbb{C}$. Prove that $n(\gamma, a)$ is constant in each component of $\mathbb{C} - \gamma$.

   b) Evaluate the integral $\int_\gamma \frac{(e^z - e^{-z})dz}{z^n}$, where n is a positive integer and $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$.

9. State and prove Goursat's theorem.

Unit – II

10. a) Find the Laurent series expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in ann(0, 1, 2).

    b) Let z = a be an isolated singularity of f and let $f(z) = \sum_{-\infty}^{\infty} a_n (z-a)^n$ be its Laurent expansion in ann(a, 0, R). Prove that z = a is a pole of order m if and only if $a_m \ne 0$ and $a_n = 0$ for $n \le -(m+1)$.

11. a) Evaluate the integral $\int_0^\pi \frac{d\theta}{a + \cos\theta}$, $a > 1$ by the method of residues.

    b) State and prove Argument principle.

12. Let $D = \{z : |z| < 1\}$ and suppose f is analytic on D with $|f(z)| \le 1$ for z in D and $f(0) = 0$. Prove that $|f'(0)| \le 1$ and $|f(z)| \le |z|$ for all z in the disk D.

Unit – III

13. a) Prove that $C(G, \Omega)$ is a complete metric space.

    b) If $\mathcal{F} \subset C(G, \Omega)$ is equicontinuous at each point of G. Prove that $\mathcal{F}$ is equicontinuous over each compact subset of G.

14. a) State and prove Hurwitz's theorem.

    b) Let $\text{Re}\, z_n > -1$, prove that the series $\sum_{n=1}^{\infty} \log(1 + z_n)$ converges absolutely if and only if the series $\sum_{n=1}^{\infty} z_n$ converges absolutely.

15. State and prove Weierstrass Factorization theorem.

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