K20P 1190

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.)
Examination, October 2020
(2017 Admission Onwards)
MATHEMATICS
MAT 3E 01 : Graph Theory

Time : 3 Hours   Max. Marks : 80

PART – A

I. Answer **any 4** questions. **Each** question carries **4** marks :

1) Prove that a set $S \subseteq V$ is an independent set of G if and only if V\S is a covering of G.

2) Define k-vertex colouring and chromatic number.

3) Every simple planar graph has a vertex of degree at most 5.

4) A graph G is embeddable in the plane if and only if it is embeddable on the sphere.

5) Let M be a matching and K be a covering such that $|M| = |K|$. Then prove that M is a maximum matching and K is a minimum covering.

6) Prove that Let l be a feasible vertex labelling of G. If G_l contains a perfect matching M^*, then M^* is an optimal matching of G. **(4×4=16)**

PART – B

Answer **any 4** questions without omitting **any** Unit. **Each** question carries **16** marks :

Unit – I

II. a) Prove that if $\delta > 0$, $\alpha' + \beta' = \nu$.

b) If G is a connected simple graph and is neither an odd cycle nor a complete graph, then prove that $X \leq \Delta$.

P.T.O.

III. a) Prove that $r(k, k) \geq 2^{k/2}$.

b) If G is 4 – chromatic, then prove that G contains a subdivision of K_4.

IV. a) Prove that $r(k, l) \leq \binom{k+l-2}{k-1}$.

b) If the odd cycles in G are pairwise intersecting, then prove that $\chi(G) \leq 5$.

c) For any positive integer k, prove that there exists a k – chromatic graph containing no triangle.

Unit – II

V. a) Let v be a vertex of a planar graph G. Prove that G can be embedded in the plane in such a way that v is on the exterior face of the embedding.

b) Let G be a connected graph that is not an odd cycle. Then prove that G has a 2 – edge colouring in which both colours are represented at each vertex of degree at least two.

c) If G is nonplanar, then prove that at least one of H_1 and H_2 is also nonplanar.

VI. a) State and prove Euler's formula.

b) Prove that every planar graph is 5 – vertex-colourable.

VII. Prove that a graph is planar if and only if it contains no subdivision of K_5 or $K_{3,3}$.

Unit – III

VIII. a) Prove that a matching M in G is a maximum matching if and only if G contains no M – augmenting path.

b) State and prove the Max-Flow, Min-cut theorem.

IX. Prove that G has a perfect matching if and only if $O(G - S) \leq |S|$ for all $S \subset V$.

X. State and prove Menger's theorem. **(4×16=64)**
