K23U 2369

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/Improvement) Examination, November 2023
(2019 – 2021 Admissions)
CORE COURSE IN MATHEMATICS
5B09MAT: Vector Calculus

Time : 3 Hours Max. Marks : 48

PART – A
(Short Answer Questions)

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Find the parametric equation for the line through (3, –4, –1) parallel to the vector v = i + j + k.

2. Find the distance from the point (2, –3, 4) to the plane x + 2y + 2z = 13.

3. Find the gradient of the function $f(x, y) = xy^2$ at the point (2, –1).

4. Evaluate $\int_C (x + y)ds$, where C is the straight line segment x = t, y = 1 – t, z = 0 from (0, 1, 0) to (1, 0, 0).

5. Define Divergence Theorem.

PART – B
(Short Essay Questions)

Answer **any eight** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find the length of the portion of the curve r(t) = 4cost i + 4sint j + 3t k, $0 \le t \le \frac{\pi}{2}$.

7. Find the curvature of r(t) = 3sint i + 3cost j + 4t k.

P.T.O.

8. Find the directions in which $f(x,y)=\frac{x^2}{2}+\frac{y^2}{2}$ increases more rapidly at (1, 1).

9. Find the plane tangent to the surface $z = x\cos y - ye^x$ at (0, 0, 0).

10. Find the work done by the force field $F = xi + yj + zk$ in moving an object along the curve C parametrized by $r(t) = \cos(\pi t)i + t^2 j + \sin(\pi t)k$, $0 \le t \le 1$.

11. Find the scalar potential of the vector field $F = 2xi + 3yj + 4zk$.

12. Find the Curl of $F = (x^2 - z)i + xe^z j + xyk$.

13. Find the critical points of the function $f(x, y) = x^2 + y^2 - 4y + 9$.

14. Find the Divergence of the vector field $F = (y^2 - x^2)i + (x^2 + y^2)j$.

15. Integrate $G(x, y, z) = x^2$ over the cone $z=\sqrt{x^2+y^2}, 0 \le z \le 1$.

16. Evaluate $\int_C y^2dx + x^2dy$, $C : x^2 + y^2 = 4$.

PART – C

(Essay Questions)

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. Find the angle between the planes $2x + 2y + 2z = 3$, $2x - 2y - z = 5$.

18. Find the unit tangent vector of the curve $r(t) = \sin t\, i + (3t^2 - \cos t)j + e^t k$, at $t_0 = 0$.

19. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at (1, 1, 0) in the direction of $v = 2i - 3j + 6k$.

20. Verify Green's theorem for $F = -yi + xj$ over the circle $C : a\cos t\, i + a\sin t\, j$, $0 \le t \le 2\pi$.

21. Verify Divergence theorem for $F = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.

22. Find the linearization $L(x, y, z)$ of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point (2, 1, 0).

23. Integrate $G(x, y, z) = xyz$ over the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$, $z = 1$.

PART – D

(Long Essay Questions)

Answer **any two** questions from this Part. Each question carries 6 marks. **(2×6=12)**

24. Find the curvature and torsion of the curve

    $r(t) = (\cos t + t\sin t)i + (\sin t - t\cos t)j,\ t > 0$.

25. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

26. Show that $ydx + xdy + 4dz$ is exact and evaluate the integral

    $\int ydx + xdy + 4dz$ over any path from (1, 1, 1) to (2, 3, –1).

27. Find the center of mass of a thin hemispherical shell of radius a and constant density δ.