Reg. No. : ....

Name : ....

K18U 0122

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Imp.) Examination, May 2018
CORE COURSE IN MATHEMATICS
6B11 MAT : Numerical Methods and Partial Differential Equations
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. If the Newton-Raphson method is used on $f(x) = x^3 - x + 1$ starting with $x_0 = 1$, what will x_1 be ?

2. Find an interval of unit length which contains the smallest positive root of the equation $x^3 - 5x - 1 = 0$.

3. The divided differences are symmetric with respect to their arguments. True or false ?

4. Give the two-dimensional Poisson equation.

(1×4=4)

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Perform three iterations of the bisection method to obtain a real root of the equation $x^3 - 6x - 4 = 0$.

6. Find the root of the equation $2x - \log_{10} x = 7$ which lies between 3.5 and 4 by Regulafalsi method.

7. Using Newton-Raphson method, derive formula to find $N^{1/q}$, $N > 0$, q integer and hence find $(18)^{1/3}$.

P.T.O.

8. Find the Lagrange interpolating polynomial that fits the following data.

x	0	1	2
f(x)	2	1	12

9. Using divided differences, show that the data

x	–3	–2	–1	1	2	3
f(x)	18	12	8	6	8	12

represents a second degree polynomial. Hence determine the interpolating polynomial.

10. The following data represents e^{-x}.

x	–1	–0.5	0	1
f(x)	2.7183	1.6487	1	0.3679

Obtain an approximate value of $f''(-1)$ using the method $f''(x_0) = [f_0 - 2f_1 + f_2] / h^2$ with (i) h = 1 and (ii) h = 1/2.

11. Find the solution of the initial value problem $y' = x - y$, $y(0) = 1$, by performing three iterations of the Picard's method.

12. Show that the wave equation $u_{tt} = c^2 u_{xx}$ is hyperbolic.

13. Find solution u(x, y) by separating variables : $xu_{xy} + 2yu = 0$.

14. Solve the system $u_{xx} = 0$, $u_{yy} = 0$. **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. The equation $x^3 - 5x - 1 = 0$ has a root in the interval (– 1, 0). Write this equation in an equivalent form $x = \phi(x)$ so that the general iteration method $x_{k+1} = \phi(x_k)$ is convergent. Hence, perform three iterations of this method starting with $x_0 = -0.5$.

16. For the data,

x	1	1.1	1.2	1.3	1.4
f(x)	7.0	8.093	9.384	10.891	12.632

find an approximation of f(1.35) and f(1.25).

17. Evaluate $\int_0^2 \frac{dx}{x^2 + 2x + 10}$ using Simpson's rule with n = 2, 4.

18. Obtain the approximate value of y(1.3) for the initial value problem $y' = -2xy^2$, y(1) = 1 using Taylor series second order method with step size h = 0.1.

19. Find the deflection u(x, t) of the string of length L = π and $c^2 = 1$ for zero initial displacement and triangular initial velocity

$$u_t(x, 0) = \begin{cases} 0.01x & \text{if } 0 \le x \le \pi/2 \\ 0.01(\pi - x) & \text{if } \pi/2 \le x \le \pi \end{cases}$$

20. Obtain the Laplacian in Polar Coordinates. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. The following table of the function $f(x) = e^{-x}$ is given.

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8
f(x)	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

a) Using Gauss backward central difference formula compute f(0.45).

b) Using Stirling central difference formula compute f(0.55).

22. Evaluate the integral $\int_0^{\pi/2} e^{-x}$ cosxdx using the trapezoidal rule with (i) h = $\pi/2$ and (ii) h = $\pi/4$.

23. Solve the initial value problem $y' = x(y - x)$, y(2) = 3 in the interval [2, 2.4] using the classical Runge-Kutta fourth order method with step size h = 0.2.

24. Derive the Fourier series solution to the heat equation. **(6×2=12)**