



K18U 0122

Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Imp.) Examination,
May 2018

CORE COURSE IN MATHEMATICS

6B11 MAT : Numerical Methods and Partial Differential Equations
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

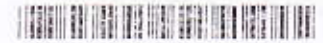
1. If the Newton-Raphson method is used on $f(x) = x^3 - x + 1$ starting with $x_0 = 1$, what will x_1 be ?
2. Find an interval of unit length which contains the smallest positive root of the equation $x^3 - 5x - 1 = 0$.
3. The divided differences are symmetric with respect to their arguments. True or false ?
4. Give the two-dimensional Poisson equation. (1×4=4)

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Perform three iterations of the bisection method to obtain a real root of the equation $x^3 - 6x - 4 = 0$.
6. Find the root of the equation $2x - \log_{10} x = 7$ which lies between 3.5 and 4 by Regula-falsi method.
7. Using Newton-Raphson method, derive formula to find $N^{1/q}$, $N > 0$, q integer and hence find $(18)^{1/3}$.

P.T.O.



8. Find the Lagrange interpolating polynomial that fits the following data.

x	0	1	2
f(x)	2	1	12

9. Using divided differences, show that the data

x	-3	-2	-1	1	2	3
f(x)	18	12	8	6	8	12

represents a second degree polynomial. Hence determine the interpolating polynomial.

10. The following data represents e^{-x} .

x	-1	-0.5	0	1
f(x)	2.7183	1.6487	1	0.3679

Obtain an approximate value of $f''(-1)$ using the method $f''(x_0) = [f_0 - 2f_1 + f_2] / h^2$ with (i) $h = 1$ and (ii) $h = 1/2$.

11. Find the solution of the initial value problem $y' = x - y$, $y(0) = 1$, by performing three iterations of the Picard's method.
12. Show that the wave equation $u_{tt} = c^2 u_{xx}$ is hyperbolic.
13. Find solution $u(x, y)$ by separating variables : $xu_{xy} + 2yu = 0$.
14. Solve the system $u_{xx} = 0$, $u_{yy} = 0$. (2×8=16)

SECTION – C

Answer **any 4** questions from among the questions **15 to 20**. These questions carry **4 marks each**.

15. The equation $x^3 - 5x - 1 = 0$ has a root in the interval $(-1, 0)$. Write this equation in an equivalent form $x = \phi(x)$ so that the general iteration method $x_{k+1} = \phi(x_k)$ is convergent. Hence, perform three iterations of this method starting with $x_0 = -0.5$.
16. For the data,
- | | | | | | |
|------|-----|-------|-------|--------|--------|
| x | 1 | 1.1 | 1.2 | 1.3 | 1.4 |
| f(x) | 7.0 | 8.093 | 9.384 | 10.891 | 12.632 |
- find an approximation of $f(1.35)$ and $f(1.25)$.



17. Evaluate $\int_0^2 \frac{dx}{x^2 + 2x + 10}$ using Simpson's rule with $n = 2, 4$.
18. Obtain the approximate value of $y(1.3)$ for the initial value problem $y' = -2xy^2$, $y(1) = 1$ using Taylor series second order method with step size $h = 0.1$.
19. Find the deflection $u(x, t)$ of the string of length $L = \pi$ and $c^2 = 1$ for zero initial displacement and triangular initial velocity

$$u_1(x, 0) = \begin{cases} 0.01x & \text{if } 0 \leq x \leq \pi/2 \\ 0.01(\pi - x) & \text{if } \pi/2 \leq x \leq \pi \end{cases}$$

20. Obtain the Laplacian in Polar Coordinates. (4x4=16)

SECTION – D

Answer **any 2** questions from among the questions **21 to 24**. These questions carry **6 marks each**.

21. The following table of the function $f(x) = e^{-x}$ is given.

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8
f(x)	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

- a) Using Gauss backward central difference formula compute $f(0.45)$.
- b) Using Stirling central difference formula compute $f(0.55)$.
22. Evaluate the integral $\int_0^{\pi/2} e^{-x} \cos x dx$ using the trapezoidal rule with (i) $h = \pi/2$ and (ii) $h = \pi/4$.
23. Solve the initial value problem $y' = x(y - x)$, $y(2) = 3$ in the interval $[2, 2.4]$ using the classical Runge-Kutta fourth order method with step size $h = 0.2$.
24. Derive the Fourier series solution to the heat equation. (6x2=12)