K18U 0124

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Imp.) Examination, May 2018
CORE COURSE IN MATHEMATICS
6B13 MAT : Mathematical Analysis and Topology
(2014 Admn. Onwards)

Time : 3 Hours   Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. If f is continuous on [a, b], then its indefinite integral is an antiderivative of f. True or False ?
2. Give an example of a sequence of continuous functions such that the limit function is not continuous.
3. Define the boundary point of a set A in a metric space X.
4. Give an example of an infinite class of closed sets whose union is not closed.

(1×4=4)

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. If $f \in R\,[a, b]$ and if (P_n) is any sequence of tagged partitions of [a, b] such that, $\| P_n \| \to 0$, prove that $\int_a^b f = \lim_n S\,(f, P_n)$.
6. If f is continuous on [a, b], a < b, show that there exists $c \in [a, b]$ such that we have $\int_a^b f = f(c)\,(b - a)$.

7. Applying the fundamental theorem show that there does not exist a continuously differentiable function f on [0, 2] such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for $0 \leq x \leq 2$.

8. If Σa_n is an absolutely convergent series, then show that the series $\Sigma a_n \sin nx$ is absolutely and uniformly convergent.

9. Prove that the sequence (f_n) defined by $f_n(x) = \frac{nx^2 + 1}{nx + 1}$ converges uniformly on the interval [1, 2].

10. Prove that every discrete metric space is complete.

11. Let X be a metric space and let A be a subset of X. If x is a limit point of A, show that each open sphere centered on x contains infinitely many distinct points of A.

12. Show that in any metric space, each open sphere is an open set.

13. Show that the intersection of two topologies on a nonempty set X is also a topology on X.

14. Prove or disprove : If X is a topological space which is not discrete, then no subspace of X is discrete. **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each.**

15. If $f : [a, b] \rightarrow \mathbb{R}$ is continuous on [a, b], show that $f \in R[a, b]$.

16. State and prove a necessary condition for a function $f : [a, b] \rightarrow \mathbb{R}$ to be in R [a, b]. Using the same show that the Dirichlet function is not Riemann integrable.

17. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow \mathbb{R}$. Show that f is continuous on A.

18. Let X be a metric space. Show that a subset F of X is closed if and only if its complement F is open.

19. Give an example of a set in a topological space which :

    a) is both open and closed

    b) is neither open nor closed

    c) contains a point which is not a limit point of the set

    d) contains no point which is not a limit point of the set.

20. Let X be a topological space and A an arbitrary subset of X. Show that $\overline{A} = \{x : \text{each neighborhood of } x \text{ intersects } A\}$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. State and prove the fundamental theorem of calculus. (second form)

22. State and prove the Cauchy criterion for uniform convergence.

23. State and prove the Baire's theorem.

24. a) Let X and Y be topological spaces and f a mapping of X into Y. When do you say that f is :

        i) continuous

        ii) open

        iii) a homeomorphism ?

    b) Let X be an infinite set. Show that $T = \{U \subseteq X : U = \varnothing \text{ or } X \setminus U \text{ is finite}\}$ is a topology on X. **(6×2=12)**