K18U 0121

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Imp.) Examination, May 2018
CORE COURSE IN MATHEMATICS
6B10MAT : Linear Algebra
(2014 Admn. Onwards)

Time : 3 Hours Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Prove or disprove: If there exists a linearly dependent set $\{v_1, v_2, ..., v_n\}$ in the vector space V, then dim (V) $\leq$ n.

2. What is the dimension of the vector space of all 2×2 matrices over R ?

3. There is not a linear transformation $T : R^3 \to R^2$ such that T (1, 0, 3) = (1, 1) and T (–2, 0, –6) = (2, 1). Why ?

4. Find the algebraic multiplicity of the eigenvalue of the matrix $\begin{bmatrix} 5 & 1 \\ 0 & \cdot 5 \end{bmatrix}$. **(1×4=4)**

SECTION – B

Answer **any 8** questions from among the questions 5 to 14. These questions carry **2** marks **each**.

5. Determine whether the following vectors span $\mathbb{R}^3$. Justify your answer. u = (1, 1, 1), v = (2, 3, 1), w = (3, 4, 2).

6. Let $T : F^2 \to F^2$ be the linear transformation defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$. Determine whether T is one-to-one and onto.

7. Construct a linear transformation $T : R^2 \to R^3$ such that T (1, 1) = (1, 0, 2) and T (2, 3) = (1, –1, 4).

P.T.O.

8. Find the eigenvalues of the matrix $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

9. True or False ? Justify : If v is an eigenvector of both A and B, then it is an eigenvector of the sum A + B.

10. Show that if A is a matrix such that $A^4 = I$, then the only possible eigenvalues of A are 1, –1, i and –i.

11. Determine the null space of the (a) Zero matrix and the (b) Identity matrix.

12. True or False ? Justify : If A is a 5×6 matrix of rank 4, then the nullity of A is 1.

13. Determine whether $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \in M_{2\times 2}(R)$ is diagonalizable or not.

14. Using Gauss elimination, solve :

$2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16.$ **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Suppose that $\{v_1, v_2, v_3\}$ is a linearly independent subset of a vector space V. Show that $\{v_1, v_1 + v_2, v_1 + v_2 + v_3\}$ is also linearly independent.

16. a) Let α be a scalar and u be a vector in a vector space V. If $\alpha u = 0$ then show that either $\alpha = 0$ or $u = 0$.

    b) Prove that if $u \neq 0$ and $\alpha u = \beta u$ in a vector space V, then $\alpha = \beta$.

17. Let V and W be vector spaces and $T : V \to W$ be linear. Show that N (T) and R (T) are subspaces of V and W respectively.

18. Find the characteristic roots and the corresponding characteristic vectors of the matrix , $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

19. Find a basis of the solution space of the following system of equations. $x + y - z + t = 0$, $x - y + 2z - t = 0$, $3x + y + t = 0$.

20. Let T be the linear operator on $P_2(R)$ defined by $T(f(x)) = f'(x)$. Show that T is not diagonalizable. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Let V be a vector space with dimension n. Prove the following :
   a) Any finite generating set for V contains at least n vectors and a generating set for V that contains exactly n vectors is a basis for V.
   b) Any linearly independent subset of V that contains exactly n vectors is a a basis for V.
   c) Every linearly independent subset of V can be extended to a basis for V.

22. Let $h(x) = 3 - 2x + x^2$. Let $U : P_2(R) \to R^3$ be the linear transformation defined by $U(a + bx + cx^2) = (a + b, c, a - b)$. Let β and γ be the standard ordered bases of $P_2(R)$ and R^3 respectively. Compute $[U]_\beta^\gamma$, $[h(x)]_\beta$ and $[U(h(x))]_\gamma$ and verify that $[U]_\beta^\gamma [h(x)]_\beta = [U(h(x))]_\gamma$.

23. Find all the values of a and b so that the following system of equations has
   i) no solution
   ii) a unique solution and
   iii) infinitely many solutions.

   $x - y + 2z = 4$, $3x - 2y + 9z = 14$, $2x - 4y + az = b$.

24. Using Gauss elimination method, find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$.

(6×2=12)
