K18U 0123

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Imp.) Examination, May 2018
CORE COURSE IN MATHEMATICS
6B12 MAT : Complex Analysis
(2014 Admn. Onwards)

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Represent the complex number $1 + i$ in the exponential polar form.

2. Evaluate $\int_{8+\pi i}^{8-3\pi i} e^{z/2}\, dz$.

3. Show that the condition, the domain be simply connected, is quite essential in Cauchy's integral theorem.

4. When do we say that f has a singularity at a point z_0 ? **(1×4=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Simplify $\frac{5i}{(1-i)\,(2-i)\,(3-i)}$ to a real number.

6. Determine whether the function f defined by

$$f(z) = \begin{cases} \frac{Re(z^2)}{|z|^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$ is continuous at $z = 0$.

7. Determine whether the function f defined by $f(z) = Im(z^2)$ is analytic.

P.T.O.

8. Show that $f(z) = \overline{z}$ does not have a derivative at any point.

9. Let $z_1 = -2 + 2i$ and $z_2 = 3i$. Find Arg $(z_1 z_2)$ and Arg (z_1/z_2).

10. Evaluate $\int_C \overline{z}\, dz$, C the parabola $y = x^2$ from $-1 + i$ to $1 + i$.

11. Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ is convergent or divergent.

12. Find the radius of curvature of the power series, $\sum_{n=0}^{\infty} \frac{n^n}{n!} (z + 2i)^n$.

13. State Laurent's theorem.

14. Find the residues at the singular points of $\frac{z^4}{z^2 - iz + 2}$. **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Find all values of $(-8i)^{1/3}$ and plot them.

16. Integrate $g(z) = \frac{z^2 + 1}{z^2 - 1}$ counterclockwise around the circle $|z - 1| = 1$.

17. Show that if f is analytic inside and on a simple closed curve C and z_0 is not on C, then $\oint_C \frac{f'(z)\, dz}{z - z_0} = \oint_C \frac{f(z) dz}{(z - z_0)^2}$.

18. Show that $\text{Ln} \frac{1+z}{1-z} = 2\left(z + \frac{z^3}{3} + \frac{z^5}{5} + \ldots\right)$.

19. If a series $z_1 + z_2 + \ldots$ is given and we can find a convergent series $b_1+b_2+\ldots$ with non-negative real terms such that $|z_n| \leq b_n$ for $n = 1, 2, \ldots$ then show that the given series converges absolutely.

20. Find the Laurent series of $\frac{e^z}{z(1-z)}$ that converges for $0 < |z-1| < R$ and determine the precise region of convergence. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. a) Is $u = xy$ a harmonic function ? If yes, find a corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

    b) Find the principal value of $(-1)^{1-2i}$.

22. a) Integrate $f(z) = z^{-2} \tan \pi z$ around any contour C enclosing 0 counter clockwise.

    b) State and prove Morera's theorem.

23. Develop $\cosh(z - \pi i)$ in a Taylor series with πi as center. Find the radius of convergence.

24. Evaluate the following integral counterclockwise.

$$\oint_C \frac{z-23}{z^2-4z-5}\,dz,\ C : |z-2| = 4.$$

(6×2=12)