K17U 0368

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Regular) Examination, May 2017
(2014 Admn.)
CORE COURSE IN MATHEMATICS
6B11 MAT : Numerical Methods and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. State the intermediate value theorem.

2. When do you say that the root of an equation is simple ?

3. Lagrange interpolating polynomial possesses permanence property. True or False.

4. Give the two-dimensional Laplace equation. **(1×4=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks each.

5. Find a real root of equation $x^3 - x - 11 = 0$ by Bisection method.

6. Perform three iterations of the regula-falsi method to obtain the smallest positive root of $x^3 - 5x + 1 = 0$.

7. Using Newton-Raphson method, derive formula to find 1/N, N > 0 and hence find 1/18.

8. Find the Lagrange interpolating polynomial that fits the following data :

x :	1	2	4
f(x) :	1	7	61

9. Construct the divided difference table for the following data :

x :	–1	0	3
f(x) :	–4	–5	16

Determine the approximate value of f(1) using divided difference interpolation.

P.T.O.

10. The following data is given :

x :	-3	-2.5	-2	-1	1
f(x) :	-25	-14.125	-7	-1	-1

Using the method $f'(x_0) = [-3f_0 + 4f_1 - f_2]/(2h)$, obtain an approximate value of $f'(-3)$ with

i) $h = 2$ and

ii) $h = 1$.

The exact value of $f'(-3)$ is 26.

11. Find the solution of the initial value problem $y' = 2y - x$, $y(0) = 1$, by performing three iterations of the Picard's method.

12. If a steel wire 2 meters in length weighs 0.8 nt (about 0.18 *l*b) and is stretched by a tensile force of 200 nt (about 45 *l*b), what is the corresponding speed c of transverse waves ?

13. Find the solution u(x, y) by separating variables :

$$y^2u_x - x^2u_y = 0.$$

14. Find the solution u(x, y) of $u_{xx} + 9u = 0$. **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each** :

15. Evaluate $\sqrt{5}$ using the equation $x^2 - 5 = 0$ by applying the fixed point iteration method.

16. Prove the following relations :

a) $\Delta\left(\frac{1}{f_i}\right) = -\frac{\Delta f_i}{f_i f_{i+1}}$

b) $\Delta\left(\frac{f_i}{g_i}\right) = -\frac{g_i\Delta f_i - f_i\Delta g_i}{g_i g_{i+1}}$

c) $\Delta\left(f_i^2\right) = (f_i + f_{i+1})\,\Delta f_i$

d) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$.

17. Evaluate $\int_0^1 \frac{dx}{3+2x}$ using Simpson's rule with n = 2, 4.

18. In the following initial value problem, find the approximate value of y(x) at the given point using the Euler method :

$y' = xy + x^2y^2 + 1, y(1) = 2, h = 0.1, x \in [1, 1.3]$.

19. Show that $\nabla^2 u$ is invariant under translations $x^* = x + a$, $y^* = y + b$ and under rotations $x^* = x\cos\alpha - y\sin\alpha$, $y^* = x\sin\alpha + y\cos\alpha$.

20. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is

$$f(x) = \begin{cases} x & \text{if } 0 < x < L/2 \\ L - x & \text{if } L/2 < x < L \end{cases}.$$

(4×4=16)

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. The following table of the function $f(x) = e^{-x}$ is given

x :	0.2	0.3	0.4	0.5	0.6	0.7	0.8
f(x) :	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

a) Using Gauss forward central difference formula compute f(0.55).

b) Using Gauss backward central difference formula compute f(0.45).

22. Evaluate $\int_1^2 \cos x dx$ using the trapezoidal rule with

i) h = 1 and ii) h = 1/2.

Compare with the exact solution.

23. In the following initial value problem, find the approximate value of y(x) at the given point using classical fourth order Runge-Kutta method.

$y' = x^2 + y^2, y(1) = 2, h = 0.1, x \in [1, 1.2]$

24. Derive the D'Alembert's solution of the wave equation. **(6×2=12)**