K17U 0370

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Regular) Examination, May 2017
CORE COURSE IN MATHEMATICS
(2014 Admn.)
6B13 MAT : Mathematical Analysis and Topology

Time : 3 Hours | Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example of a function $f:[0,1]\to\mathbb{R}$ that is in R [c, 1] for every $c\in(0,1)$ but which is not in R [0, 1].

2. Find $\lim\limits_{n\to\infty}\frac{x^n}{1+x^n}$ for $x\in\mathbb{R}, x\geq 0$.

3. Let d be the discrete metric on a set X which contains at least two points. Then for $x\in X$, what is the diameter of the open sphere $S_{1/2}(x)$?

4. Give an example of a Cauchy sequence in a metric space X that does not converge in X. **(1×4=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. If f ∈ R [a, b] and $|f(x)|\leq M$ for all $x\in[a, b]$ show that $\left|\int_a^b f\right|\leq M(b-a)$.

6. If f ∈ R [a, b] show that F defined by, $F(z)=\int_a^z f$ for $z\in[a, b]$, is continuous on [a, b].

P.T.O.

7. If f and g belong to R [a, b] show that the product fg belongs to R [a, b].

8. Show that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

9. State and prove Weierstrass M-test for uniform convergence.

10. Let X be a metric space. Show that every subset of X is open $\Leftrightarrow$ each subset of X which consists of a single point is open.

11. Write a short note on the Cantor set.

12. Show that in any metric space, each closed sphere is a closed set.

13. Show that the union of two topologies on a nonempty set X need not be a topology on X.

14. Prove or disprove : If A and B are subsets of a topological space X with $\overline{A} = \overline{B}$, then A = B. **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. If $f \in R[a, b]$, show that f is bounded on [a, b].

16. Suppose that f is continuous on [a, b], that $f(x) \geq 0$ for all $x \in [a, b]$ and that $\int_a^b f = 0$. Prove that f(x) = 0 for all $x \in [a, b]$. Can the continuity hypothesis be dropped ? Justify.

17. Let $f_n(x) = \frac{nx}{1 + nx}$ for $x \in [0, 1]$.

a) Evaluate $\lim_{n \to \infty} \int_0^1 f_n(x)\, dx$

b) Find the pointwise limit function f

c) Evaluate $\int_0^1 f(x)\, dx$

d) Does (f_n) converge uniformly to f ?

18. Let X be a metric space with metric d. Show that d_1 defined by

$$d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$$ is also a metric on X.

19. Let X be a topological space and A a subset of X. Show that

    i) $\overline{A} = A \cup D(A)$ and

    ii) A is closed $\Leftrightarrow A \supseteq D(A)$

20. Let X = {1, 2, 3} and with the topology T = {X, ∅, {1}, {1, 2}, {1, 3}}.

    a) List all closed subsets of X

    b) Find the closure of {1}

    c) Find the closure of {2}

    d) List all open subsets of X

(4×4=16)

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. State and prove the fundamental theorem of calculus (first form).

22. Show that the uniform convergence of the sequence of continuous functions is sufficient to guarantee the continuity of the limit function. Is it necessary ? Justify.

23. State and prove Cantor's intersection theorem.

24. State the Kuratowski closure axioms on a non-empty set X and show that it defines a topology on X.

(6×2=12)