K17U 0367

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Regular) Examination, May 2017
CORE COURSE IN MATHEMATICS
6B10MAT : Linear Algebra
(2014 Admn.)

Time : 3 Hours   Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give example of an infinite dimensional vector space.
2. Give a basis for the vector space of complex numbers over the field of real numbers.
3. Find the matrix of the reflection about the x – axis with respect to the standard basis of $\mathbb{R}^2$.
4. Is the matrix $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ diagonalizable ? Justify. **(1×4=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Determine whether $\{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1\}$ is a subspace of $\mathbb{R}^3$ or not.
6. Give an example of distinct linear transformations T and U such that N(T) = N(U) and R(T) = R(U).
7. Let V and W be vector spaces and $T : V \to W$ be linear. Show that T is one-to-one if and only if N(T) = {0}.
8. If A is a 4×9 matrix, what is the smallest possible value for nullity (A) ?
9. True or False ? Justify.
   If X is a nontrivial solution of AX = 0, then every entry in X is nonzero.

10. State and prove Sylvester's law of nullity.

11. Prove or disprove : If λ is an eigenvalue of both A and B, then it is an eigenvalue of the sum A + B.

12. Find the eigenvalues of the matrix $F_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$.

13. Show by example that a diagonalizable linear operator on an n-dimensional vector space need not possess n distinct eigenvalues.

14. Use Gaussian elimination to solve the system of linear equations

$x_1 - 2x_2 - 6x_3 = 12,\ 2x_1 + 4x_2 + 12x_3 = -17,\ x_1 - 4x_2 - 12x_3 = 22.$ **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. Is the set of all differentiable real-valued functions defined on R a subspace of C(R) ? Justify your answer.

16. Let S be a linearly independent subset of the vector space V and let $v \in V \setminus S$. Show that $S \cup \{v\}$ is linearly dependent if and only if $v \in \text{span}(S)$.

17. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that T(1, 1) = (1, 3) and T(–1, 1) = (3, 1). Find T(a, b).

18. Suppose that AX = B has a solution. Show that this solution is unique if and only if AX = 0 has only the trivial solution.

19. Find the characteristic roots and the corresponding characteristic vectors of the matrix, $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

20. Using Gauss elimination method, find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Let S be a linearly independent subset of a vector space V. Show that there exists a maximal linearly independent subset of V that contains S.

22. Let $g(x) = 3 + x$. Let $T : P_2(R) \to P_2(R)$ and $U : P_2(R) \to R^3$ be the linear transformations respectively defined by

$T(f(x)) = f'(x)g(x) + 2f(x)$ and $U(a + bx + cx^2) = (a + b, c, a - b)$.

Let β and γ be the standard ordered bases of $P_2(R)$ and R^3 respectively. Compute

$[U]_\beta^\gamma$, $[T]_\beta$, $[UT]_\beta^\gamma$ directly and verify that $[UT]_\beta^\gamma = [U]_\beta^\gamma [T]_\beta$.

23. Investigate for what values of λ, μ the system of simultaneous equations, $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$, has (i) no solution, (ii) a unique solution (iii) infinitely many solutions.

24. Let T be the linear operator on R^3 defined by $T\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 4a_1 + a_3 \\ 2a_1 + 3a_2 + 2a_3 \\ a_1 + 4a_3 \end{pmatrix}$.

a) Find the eigenvalues of T and their multiplicities.

b) Determine the eigenspaces corresponding to these eigenvalues.

c) Show that T is diagonalizable. **(6×2=12)**