K17U 0115

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, May 2017
CORE COURSE IN MATHEMATICS
6B12 MAT : Linear Algebra (2009-2013 Admns.)

Time : 3 Hours Weightage : 30

1. Fill in the blanks :
   a) The number of elements in the basis of a vector space is ________.
   b) Example for a subspace of $\mathbb{R}^2$ is ________.
   c) In the system of equations AX = B, if row-rank (A) = row-rank (AB) < number of unknowns, then the number of solutions is ________.
   d) If T is a linear transformation, then the value of T(0) is ________.

(Weightage 1)

Answer **any six** from the following **(Weightage 1 each).**

2. What do you mean by linear combination of vectors ?
3. Prove that intersection of two subspaces of a vector space is also a subspace of the vector space.
4. Prove that every subset of a linearly independent set is also linearly independent.
5. Using graphs, solve $2x + y = 3$; $x - 2y = -1$.
6. Compare row-echelon form and row-reduced echelon form of a matrix.
7. Find the characteristic polynomial of $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$.
8. Give an example for a linear transformation.
9. What do you mean by rank and nullity of a linear transformation ?
10. Define row-rank, column-rank and rank of a matrix. **(Weightage 1×6=6)**

Answer **any seven** from the following **(Weightage 2 each).**

11. Prove that $V = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 / x_1 + x_5 = 0\}$ is a vector space with respect to usual addition and scalar multiplication of vectors.

12. Determine whether the set $\{1 + x, x + x^2, x^2 + 1\}$ of vector space of polynomials of degree ≤ 2 is linearly independent or not.

13. Test for consistency and solve the equations $2x + 3y + 2z = 16$, $3x + y + z = 6$, $x + 5y + 3z = 1$.

14. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

15. Prove that for a symmetric matrix any two eigen vectors from different eigen spaces are orthogonal.

16. Prove that constant term of the characteristic polynomial of a matrix A is $(-1)^n|A|$ where n is the order of A.

17. Check whether $T : R^2 \to R^2$ defined by $T(1, 2) = (2, 3)$, $T(0, 1) = (1, -1)$, $T(3, -4) = (5, 7)$ is linear.

18. Find the null space, range space and their dimensions of the linear transformation $T : R^3 \to R^4$ defined by $T(x, y, z) = (x, x + y, x + y + z, z)$.

19. Let T be a linear operator from $\mathbb{R}^3$ to P_2, the set of all polynomials of degree ≤ 2 defined by $T(a, b, c) = (a + b) + (b + c)x + (c + a)x^2$. Prove that T is one-one and onto and hence find T^{-1}.

20. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. **(Weightage 2×7=14)**

Answer **any three** from the following **(Weightage 3 each).**

21. Let $U = \{(x_1, x_2, x_3) \in \mathbb{R}^3 / 2x_1 - 3x_2 + 5x_3 = 0\}$ and $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 / 4x_1 + x_2 - 3x_3 = 0\}$ be subspaces of $\mathbb{R}^3$. Find a basis and dimension of U, W and $U \cap W$.

22. Using row elementary transformations, find the inverse of the matrix $\begin{bmatrix} -1 & 1 & 1 \\ 3 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$.

23. Show that the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ satisfies its characteristic equation. Also find its inverse.

24. Diagonalise the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

25. Let $T : R^3 \to P_2$ be a linear map and matrix corresponding to the linear map T be $\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & -2 \\ 1 & -1 & 3 \end{bmatrix}$ where $B_1 = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}$ is a basis of R^3 and $B_1 = \{1 + x, x + x^2, x^2 + 1\}$ is a basis of P_2. Find T(x, y, z) for a vector $(x, y, z) \in R^3$.

(Weightage 3×3=9)
