K17U 0116

Reg. No. : .....

Name : .....

VI Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, May 2017
CORE COURSE IN MATHEMATICS
6B13 MAT : Integral Transforms
(2009-2013 Admns.)

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :
   a) Laplace transform of t^2 is _____
   b) Fundamental period of cos2x is _____
   c) Product of an even and an odd function is _____
   d) Z (u(n)) = _____ **(Weightage : 1)**

Answer **any six** from the following. (Weightage **1 each**) :

2. State the condition for the existence of Laplace transform.
3. Find $L((t+1)^2e^t)$.
4. Find the inverse Laplace transform of $\frac{2}{s^2+s+\frac{1}{2}}$.
5. Explain the Fourier sine series and cosine series expansion of functions.
6. State final value theorem for Z-transform.
7. Find Z-transform of $\cos\frac{n\pi}{2}$.
8. Find Z-transform of $(t + T)e^{-(t+T)}$.
9. Explain Fourier integral representation of functions.
10. Find Fourier cosine transform of $f(x) = e^{-x}$. **(Weightage : 6×1=6)**

Answer **any seven** from the following. (Weightage **2 each**) :

11. State and prove first shifting theorem for Laplace transform.
12. Find the inverse Laplace transform of $\log\left(1+\frac{w^2}{s^2}\right)$.

P.T.O.

13. Find the Fourier series expansion of $f(x) = \pi x$ in the interval $0 < x < 2$.
14. Express $f(x) = t - t^2$ as a half range sine series in $0 < t < 1$.
15. Find the complex Fourier series of $f(x) = e^{-x}$, $-1 \le x \le 1$.
16. State and prove first shifting theorem for Z-transforms.
17. Find the Z-transform of $f * g$ where $f(n) = \cos\frac{n\pi}{2}$ and $g(n) = \sin\frac{n\pi}{2}$.
18. Using convolution method, find the inverse Z-transform of $\frac{z^2}{(z-2)(z-3)}$.
19. Find the Fourier integral of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$.
20. With usual notation, prove that $F_s\{f''(x)\} = -w^2 F_s\{f(x)\} + \sqrt{\frac{2}{\pi}}\, wf(0)$.

(Weightage : 7×2=14)

Answer **any three** from the following. (Weightage **3 each**) :

21. Solve the integral equation $y(t) = t + \int_0^t y(\tau)\sin(t-\tau)\,d\tau$.
22. Using Laplace transform, solve the initial value problem :

    $\frac{dy}{dt} + 2x + y = 0$; $\frac{dx}{dt} + 5x - 2y = t$, $y(0) = x(0) = 0$.
23. Obtain the Fourier series for the function $f(x) = x - x^2$ in the interval $(-\pi, \pi)$.

    Deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots = \frac{\pi^2}{12}$.
24. a) Find Z-transform of $\cos n\theta$.

    b) Find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z-1)}$.
25. Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| < 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$.

(Weightage : 3×3=9)