K16U 0202

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2016
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis

Time : 3 Hours

Max. Weightage : 30

1. Fill in the blanks (weightage **1**) :

   a) If $z_1 = 8 + 3i$ and $z_2 = 9 - 2i$, then $z_1/z_2 =$ ___________

   b) If a function $f : \mathbb{C} \to \mathbb{C}$ is continuous at z_0, then $\lim_{z \to z_0} f(z) =$ __________

   c) The singularities of $\frac{1}{\sin\left(\pi/z\right)}$ are ___________

   d) If $f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n$, then residue of f(z) at z = a is __________ **(W = 1)**

Answer **any six** questions from the following **nine** questions (weightage **one each**).

2. Reduce the quantity $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ to a real number.

3. Show that $|z - 1 + 3i| = 2$ represents a circle, find its centre and radius.

4. Show that $f(z) = \bar{z}$ is not differentiable, where $z = x + iy$.

5. Show that $u(x, y) = \frac{y}{x^2 + y^2}$ is harmonic.

6. Find the values of z such that $e^z = 1$.

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7. Evaluate $\int_C \frac{z}{(9-z^2)(z+i)}\,dz$.

8. Prove that $\sin^{-1}(z) = -i \log\left[iz + (1-z^2)^{1/2}\right]$.

9. State the Cauchy's residue theorem.

10. Find the residue of $f(z) = \frac{z}{(z-1)(z+1)^2}$ at the poles. **(6×1=6)**

Answer **any 7** questions from the following **10** questions (weightage **2 each**).

11. Prove that an analytic function of constant absolute value is a constant.

12. Show that $u(x, y) = y^3 - 3x^2y$ is harmonic and find its harmonic conjugate.

13. If w(t), a complex valued function of a real variable, is integrable on [a, b], show that $\left|\int_a^b w(t)dt\right| \le \int_a^b |w(t)|dt$.

14. Find all the values of $(-8i)^{1/3}$.

15. Find $\int_C z^{1/2}dz$, where $z = 3e^{i\theta}$, $0 \le \theta \le \pi$.

16. Find the Laurent series of $f(z) = \frac{-1}{(z-1)(z-2)}$ in $1 < |z| < 2$.

17. If f(z) is analytic inside and on a positively oriented circle C with centre at z_0 and radius R, show that $|f^n(z_0)| \le \frac{n!M}{R^n}$ $(n = 1, 2, \ldots)$, where M is a positive real number such that $|f(z)| \le M$.

18. If $z = z_0$ is a pole of order m of an analytic function f(z), show that $f(z) = (z - z_0)^m g(z)$, where g(z) is analytic and non-zero at z_0.

19. Show that $z = \frac{\pi i}{2}$ is a simple pole of $f(z) = \frac{\tanh z}{z^2}$ and find the residue of f(z) at this pole.

20. If two functions p and q are analytic at a point z_0, $p(z_0) \neq 0$, $q(z_0) = 0$ and $q'(z_0) \neq 0$, show that z_0 is a simple pole of the quotient $\frac{p(z)}{q(z)}$ and also prove that $\operatorname*{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q(z_0)}$. **(7×2=14)**

Answer **any 3** questions from the following **5** questions (weightage **3 each**).

21. If $f(z) = u(x, y) + iv(x, y)$, $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$, show that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x, y) \to (x_0, y_0)} u(x, y) = u_0$ and $\lim_{(x, y) \to (x_0, y_0)} v(x, y) = v_0$.

22. If $f(z) = u(x, y) + iv(x, y)$ is defined throughout some ε-neighbourhood of $z_0 = x_0 + iy_0$, u_x, u_y, v_x, v_y exist and are continuous everywhere in this neighbourhood and u and v satisfy the Cauchy-Riemann equations at (x_0, y_0), show that $f'(z_0)$ exists.

23. State and prove Cauchy's integral formula.

24. State and prove Liouville's theorem.

25. If f(z) is analytic throughout a disk $|z - z_0| < R_0$ centred at z_0 and with radius R_0, show that f(z) has the power series representation $f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$, where $a_n = \frac{f^{(n)}(z_0)}{n!}$. **(3×3=9)**

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