K16U 0201

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improve.) Examination, May 2016
Core Course in Mathematics
6B10 MAT : ANALYSIS AND TOPOLOGY

Time : 3 Hours Max. Weightage. : 30

1. Fill in the blanks :

   a) The radius of convergence of the power series $\sum \frac{x^n}{n}$ is ________

   b) Let $A \subseteq R$ and $\phi : A \to R$ is bounded on A. Then the uniform norm of ϕ on A is $\|\phi\|_A =$ ________

   c) Let F, G be differentiable on [a, b] and let $f = F'$ and $g = G'$ belongs to R[a, b]. Then $\int_a^b fG =$ ________

   d) Let X be an arbitrary metric space and $A \subseteq X$. Then Int (A) = ________

(Weightage 1)

Answer **any six** from the following. **(Weightage 1 each)**

2. Prove the every constant function on [a, b] is R [a, b].

3. Evaluate $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\, dt$.

4. Let $G(x) = x^n (1 - x)$ for $x \in A = [0, 1]$. Prove that the convergence of $\{G(x)\}$ to 0 is uniform on A.

P.T.O.

5. Define uniform convergence of a series of functions $\sum f_n$.

6. Define closed sphere in a metric space X. Give an example.

7. Give an example of two subsets A and B of the real line such that

$(A \cup B) \neq \text{Int}(A) \cup \text{Int}(B)$.

8. Let (X, d) be a metric space and $A \subseteq X$. Define the closure of A. Prove that A is closed if and only if $A = \overline{A}$.

9. Let T_1 and T_2 be two topologies in a non-empty set X and show that $T_1 \cap T_2$ is also a topology on X.

10. Let X be a topological space and let $X \subseteq X$. Define the boundary of A and prove that it is a closed set. **(Weightage 6×1=6)**

Answer **any seven** from the following. **(Weightage 2 each)**

11. If $f \in R[a, b]$, then prove that f is bounded on [a, b].

12. If $f : [a, b] \to R$ is monotone on [a, b] then prove that $f \in R[a, b]$.

13. Prove that a sequences (f_n) of bounded functions on $A \subseteq R$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

14. Let R be the radius of convergence of $\sum a_n x^n$ and K be a closed and bounded interval contained in the interval convergence (– R, R). Then prove that the power series converges uniformly on K.

15. State and prove Dini's theorem.

16. Prove that

(a) $\lim \left(\frac{x^2 + nx}{n}\right) = x$ for $x \in R$.

(b) $\lim \left(\frac{\sin(nx + n)}{n}\right) = 0$ for $x \in R$.

17. Let X be a topological space and A an arbitrary subset of X. Then prove that

$\overline{A}$ = {x : each neighbourhood of x intersects A}.

18. Let X be a topological space and A a subset of X. Then prove that (i) $\overline{A} = A \cup D(A)$ (ii) A is closed if and only if $A \supseteq D(A)$.

19. Prove that a closed subspace of complete metric space is complete.

20. Let X be a metric space. Then prove that any intersection of closed sets in X is closed. **(Weightage 7×2=14)**

Answer **any three** from the following. **(Weightage 3 each)**

21. Let X be a complete metric space and let $\{F_n\}$ be a decreasing sequence of non-empty closed subsets of X such that $F = \cap_{n=1}^{\infty} F_n$ contains exactly one point.

22. State and prove Kuratowski's closure axioms on a topological space X.

23. Prove that a function $f : [a, b] \to R$ belongs to R[a, b] if and only if for every $\in > 0$, there exists $y_{\in} > 0$ such that if P and Q are any tagged partitions of [a, b] with $\|P\| < y_{\in}$ and $\|Q\| < y_{\in}$, then $|S(f,P) - S(f,Q)| < \in$.

24. Let (f_n) be a sequence of functions in R[a, b] and suppose that $\{f_n\}$ converges uniformly on [a, b] to f. Then prove that $f \in R[a, b]$.

25. State and prove Fundamental Theorem of Calculus (Second form). **(Weightage 3×3=9)**
