Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2015
CORE COURSE IN MATHEMATICS
6B12 MAT : Linear Algebra

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) Dimension of $\mathbb{C}$ the set of all complex numbers is ________

   b) The smallest subspace of a vector space is ______

   c) In a row reduced echelon matrix, the non-zero leading entry in a row is ________

   d) If T is a linear transformation, then the dimension of null space of T is known as __________ **(Weightage 1)**

Answer **any six** from the following (Weightage **1 each**) :

2. Define subspace of a vector space.
3. Give a basis for $\mathbb{R}$.
4. By an example, show that union of two subspaces of a vector space need not be a subspace.
5. Using graphs, solve $2x + y = 3$; $4x + 2y = -1$.
6. What do you mean by row echelon form of matrix ? Give an example.
7. State Cayley Hamilton theorem.
8. Check whether the function $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined $T(x, y, z) = (x - y, y + z)$ is a linear transformation or not.
9. Prove that range space of a linear transformation from the vector space U to V is a subspace of V.
10. What do you mean by non-singular transformation ? **(Weightage 6×1=6)**

P.T.O.

Answer **any seven** from the following (Weightage **2 each**) :

11. Let P_n be the set of all polynomials of degree $\leq n$. Let $V = \{p(x) \in P_n(x) / p(1) = 0\}$. Show that V is a vector space with respect to usual addition and scalar multiplication of polynomials.

12. Find k such that $\{(2, -1, 3), (3, 4, -1), (k, 2, 1)\}$ is linearly independent.

13. Show that the equations $x + 2y + z = 2$, $2x + y - 10z = 4$, $2x + 3y - z = 2$ are consistent and solve them.

14. Find the eigen values and eigen vector corresponding to the smallest eigen value of the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -4 & 8 & 1 \\ -1 & -2 & 0 \end{bmatrix}.$$

15. Prove that for a symmetric matrix any two eigen vectors from different eigen space are orthogonal.

16. Prove that similar matrices have the same characteristic polynomial.

17. Check whether $T : R^3 \to R^3$ defined by $T(1, 2, 2) = (2, 3, 1)$, $T(0, 1, 2) = (1, -1, 3)$, $T(3, -4, 1) = (1, 1, -2)$ and $T(3, -1, 5) = (4, 3, 2)$ is linear.

18. Let $T : R^3 \to R^3$ be a linear map defined by $T(x, y, z) = (x - y, 2y + z, 0)$. Find the null space, range space and check whether T is one-to-one.

19. Let U and V be two finite dimensional vector spaces and $T : U \to V$ be a linear map. If dim U = dim V = n, then prove that T is one-one if and only if T is onto.

20. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & -3 & -1 \end{bmatrix}$. **(Weightage 7×2=14)**

Answer **any three** from the following (Weightage **3 each**) :

21. If U and V are subspaces of a finite dimensional vector space, prove that $\dim(U + W) = \dim U + \dim V - \dim(U \cap V)$.

22. Using row elementary transformations, find the inverse of the matrix
$$\begin{bmatrix} 3 & -2 & 1 \\ 1 & 3 & -2 \\ 2 & -1 & 3 \end{bmatrix}.$$

23. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$.

24. Diagonalise the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

25. State and prove rank-nullity theorem. **(Weightage : 3×3=9)**
