Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2015
CORE COURSE IN MATHEMATICS
6 B13 MAT : Integral Transforms

Time : 3 Hours   Max. Weightage : 30

1. Fill in the blanks :

   a) Laplace transform of f (t) = 1 is _____

   b) Example for a periodic function is _____

   c) Example for an even function is _____

   d) $Z(\delta(n)) =$ _____   **(Weightage 1)**

Answer **any six** from the following : **(Weightage 1 each)**

2. Define Laplace transform.

3. Find L (sin at sinh at).

4. Find the inverse Laplace transform of $\dfrac{3}{s^2 + 6s + 18}$.

5. Explain the expressions for the Fourier coefficients a_0, a_n and b_n of a periodic function f (x) with a period 2L in the interval (– L, L).

6. Explain the convergence of Z-transform.

7. Find Z-transform of $\dfrac{1}{n}$.

8. State convolution theorem for Z-transforms.

P.T.O.

9. Explain the conditions for the existence of Fourier transform.

10. Find Fourier sine transform of $f(x) = \begin{cases} x^2 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$. **(Weightage 6×1=6)**

Answer **any seven** from the following : **(Weightage 2 each)**

11. Define unit step function. Also find Laplace transform of $f(t) = \begin{cases} t-1; & 1 < t < 2 \\ 3-t; & 2 < t < 3 \end{cases}$.

12. Find the inverse Laplace transform of $\dfrac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$.

13. Find the Fourier series expansion of $f(x) = x^2$ from $(-l, l)$.

14. Express $f(x) = x \sin x$ as a half range cosine series in $0 < x < \pi$.

15. Compute the total square error of F with $N = 3$ relative to $f(x) = x + \pi, -\pi < x < \pi$ on the interval $-\pi \le x \le \pi$.

16. State and prove initial value theorem for the Z-transforms.

17. Find the Z-transform of $f * g$ where $f(n) = n(n-1)$ and $g(n) = 3^n$.

18. Using long division method, find the inverse Z-transform of $\dfrac{10z}{z^2 - 3z + 2}$.

19. Find the Fourier sine integral of $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$.

20. Let $f(x)$ be continuous on the x-axis, $f(x) \to 0$ as $|x| \to \infty$ and $f'(x)$ be absolutely integrable on the x-axis, then prove that $F\{f'(x)\} = iw\, F\{f(x)\}$.

(Weightage 7×2=14)

Answer **any three** from the following : **(Weightage 3 each)**

21. Using convolution theorem, find the inverse Laplace transform of $\dfrac{s}{(s+a^2)^2}$.

22. Using Laplace transform, solve the initial value problem :

   $y''' + 2y'' - y' - 2y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 6$.

23. Obtain the Fourier series for the function $f(x) = |x|$, $-\pi < x < \pi$. Deduce that

   $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots = \dfrac{\pi^2}{8}$.

24. a) Find Z-transform of $r^n \cos n\theta$.

   b) Find the inverse Z-transform of $\dfrac{z}{z^2 + 7z + 10}$.

25. Deduce complex Fourier integral representation formula from the Fourier integral formula.

   (Weightage 3×3=9)

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