Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2015
CORE COURSE IN MATHEMATICS
6B10 MAT : Analysis and Topology

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

a) The Riemann sum of a function $f : [a, b] \to R$ corresponding to the partition $P = \{x_0, x_1, \ldots x_n\}$ is ____________

b) The value of $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\,dt$ is ____________

c) $\lim \frac{\sin(nx+n)}{n} =$ ____________ for $x \in R$.

d) Let X be a matrix space and let $A \subseteq X$. Then ____________ $= \{x : x \in A$ and $S_r(x) \subseteq A$ for some $r\}$. **(Weightage 1)**

Answer **any six** from the following. (Weightage **1 each**) :

2. State Cauchy's criterion for a function f to be Riemann integrable.

3. Define "Pointwise convergence" of a sequence of functions. Give an example.

4. State Dini's Theorem.

5. Define the radius of convergence of the power series $\sum a_n x^n$. Determine the radius of convergence for the series $\sum \frac{x^n}{n^2}$.

P.T.O.

6. Let $g_n(x) = x^n$ for $x \in [0, 1] = A$ and $n \in N$ and let $g(x) = 0$ for $0 \le x \le 1$ and $g(1) = 1$. Prove that the sequence $\{g_n\}$ does not converge uniformly on A.

7. Define complete metric space. Prove that (0, 1) is not complete with Euclidean metric.

8. Let X be any metric space. Prove that any intersection of open sets in X need not be open.

9. Define homeomorphism between two topological spaces. Give an example.

10. When a subset A of X is said to be nowhere dense in X ? Give an example.

(Weightage 6×1=6)

Answer **any seven** from the following. (Weightage **2 each**) :

11. If $f : [a, b] \to R$ is continuous on [a, b], then prove that $f \in R[a, b]$.

12. If $f, g \in R[a, b]$ then prove that $f g \in R[a, b]$.

13. Let (f_n) be a sequence of continuous functions on a set $A \subseteq R$ and suppose that (f_n) converges uniformly on A to a function $f : A \to R$. Then prove that f is continuous on A.

14. Let $f_n : [0, 1] \to R$ be defined for $n \ge 2$ by $f_n(x) = \begin{cases} n^2 x & \text{for } 0 \le x \le \frac{1}{n} \\ -n^2\left(x - \frac{2}{n}\right) & \text{for } \frac{1}{n} \le x \le \frac{2}{n} \\ 0 & \text{for } \frac{2}{n} \le x \le 1 \end{cases}$

Show that $\int_0^1 f(x)dx \ne \lim \int_0^1 f_n(x)\,dx$.

15. Prove that $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ for all $x \in R$.

16. Let X be an arbitrary non-empty set and defined by

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$$

Prove that d is a metric on X.

17. In any metric space X, prove that each open sphere is an open set.

18. Let X be a metric space. Prove that a subset F of X is closed if and only if its complement F' is open.

19. Let X be a topological space and A be a subset of X. Then prove that $\overline{A} = A \cup D(A)$.

20. Let X be a topological space and A an arbitrary subset of X. Then prove that $\overline{A} = \{x :$ each neighbourhood of x intersects $A\}$. **(Weightage 7×2=14)**

Answer **any three** from the following. (Weightage **3 each**) :

21. State and prove Cantor's Intersection Theorem.

22. State and prove Kuratowski closure axioms on topological space.

23. If R is the radius of convergence of the power series $\sum a_n x^n$, then prove that the series converges absolutely if $| x | < R$ and is divergent if $| x | > R$.

24. Let (f_n) be a sequence of functions in R[a, b] and suppose that (f_n) converges uniformly on [a, b] to f. Then prove that $f \in R[a, b]$.

25. State and prove Fundamental Theorem of Calculus (first form).

(Weightage 3×3=9)
