M 6057

Reg. No. : .....

Name : .....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2014
CORE COURSE IN MATHEMATICS
6B14 MAT : Operation Research (Elective – 3)

Time: 3 Hours Max. Weightage: 30

Instruction : *Answer to* ***all*** *questions.*

1. Fill in the blanks :
   a) Let S be a non-empty convex subset of R^n. Then a function f(x) on S is said to be convex if for any two vectors x_1 and x_2 in S _____
   b) Let the constraints of a general L.P.P. be $\sum_{j=1}^{n} a_{ij} x_j \le b_i$ for i = 1, 2, ... k.
   Then the non-negative variables x_{n+i} which satisfy $\sum_{j=1}^{n} a_{ij} x_j + x_{n+i} = b_i$ for i = 1, 2, ... k are called _____
   c) A necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that _____
   d) A game is said to be fair if _____ **(W : 1)**

Answer **any 6** questions from the following (**Weightage 1 each**) :

2. For a L.P.P. define :
   a) Feasible solution b) Surplus variable.
3. Express $Q(x) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$ in the form X^TAX.
4. How will you recognise optimality in the simplex method ?
5. Define the term "loop" associated with transportation problem.
6. Give the mathematical formulation of an assignment problem.
7. Define the following in game theory :
   a) Saddle point b) Optimum strategy.

P.T.O.

8. Define :
   i) Total elapsed time ii) Idle time on a machine.

9. Explain "Principle of dominance" in game theory.

10. State the "Reduction Theorem" in a assignment problem. **(W : 6×1=6)**

Answer **any 7** from the following (Weightage **2 each**) :

11. Rewrite in standard form the following linear programming problem :
    Minimize $z = 2x_1 + x_2 + 4x_3$ subject to the constraints
    $-2x_1 + 4x_2 \leq 4$, $x_1 + 2x_2 + x_3 \geq 5$, $2x_1 + 3x_3 \leq 2$
    $x_1, x_2 \geq 0$ and x_3 unrestricted in sign.

12. Show that the set $S = \{(x_1, x_2) : 3x_1^2 + 2x^2 \leq 6\}$ is convex.

13. Solve graphically the following L.P.P.
    Maximize $z = 8x_1 + 6x_2$ subject to
    $4x_1 + 2x_2 \leq 60$
    $2x_1 + 4x_2 \leq 48$
    $x_1 \geq 0$ and $x_2 \geq 0$.

14. Explain different steps involved in Simplex Algorithm.

15. Write down the dual of the following problem :
    Max. : $z = 4x_1 + 2x_2$ subject to
    $-x_1 - x_2 \leq -3$
    $-x_1 + x_2 \geq -2$; $x_1, x_2 \geq 0$.

16. Find the initial feasible solution to the transportation problem given below by North West Corner Rule.

		Destination		
Origins	D_1	D_2	D_3	**Supply**
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

17. In a factory there are 6 jobs to perform, each of which should go through machines A and B in the order A, B. The processing timings (in hours) for the jobs are given. Determine a sequence that would minimise the total elapsed time.

Job :	J_1	J_2	J_3	J_4	J_5	J_6
Machine A :	1	3	8	5	6	3
Machine B :	5	6	3	2	2	10

18. Write a note on the Least Cost method with reference to a transportation problem.

19. Solve the game whose pay off matrix is given by

Player B

$$\textbf{Player A} \begin{bmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix}$$

20. Solve the following minimal assignment problem

Man

Job	**1**	**2**	**3**	**4**
I	12	30	21	15
II	18	33	9	31
III	44	25	24	21
IV	23	30	28	14

(W : 7×2=14)

Answer **any 3** questions from the following (Weightage **3 each**) :

21. Solve the following using simplex method

Maximize $z = 6x_1 + 4x_2$

subject to

$-2x_1 + x_2 \leq 2$

$x_1 - x_2 \leq 2$

$3x_1 + 2x_2 \leq 9 \qquad x_1, x_2 \geq 0.$

22. Use dual simplex method to solve the following L.P.P.
Minimize $z = 3x_1 + x_2$
Subject to
$x_1 + x_2 \geq 1$; $2x_1 + 3x_2 \geq 2$
$x_1, x_2 \geq 0$.

23. Solve the following transportation problem by Vogel's method.

		To I	II	III	Supply
From	1	2	7	4	5
	2	3	3	1	8
	3	5	4	7	7
	4	1	6	2	14
	Demand	7	9	18	

24. Use graphical method to minimize the time added to process the following jobs on the machine shown, u, for each machine find the job which should be done first. Also calculate the total time elapsed to complete both jobs

Job 1 :	Sequence	A	B	C	D	E
	Time	3	4	2	6	2

Job 2 :	Sequence	C	B	A	D	E
	Time	5	4	3	2	6

25. Solve the following 2 × 2 game graphically

Player B

		B_1	B_2	B_3	B_4
Player A	A_1	2	1	0	−2
	A_2	1	0	3	2

(W : 3×3=9)