M 6053

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2014
CORE COURSE IN MATHEMATICS
6B12 MAT : Linear Algebra

Time: 3 Hours   Max. Weightage : 30

1. Fill in the blanks :
   a) Dimension of $\mathbb{R}^n$ is ____________
   b) Is $\mathbb{Q}$ a vector Space over $\mathbb{R}$?
   c) The system of linear equation AX = B is consistent if ____________
   d) If a matrix A is in row-reduced echelon form, non-zero leading entry in a row is ____________ **(Weightage 1)**

Answer **any six** from the following (Weightage **1 each**) :

2. Define vector space.
3. Give a basis for $\mathbb{R}^3$.
4. Prove that intersection of any number of subspaces of a vector space V is a subspace of V.
5. What do you mean by row elementary operations ?
6. If λ is an eigen value of A, prove that λ^n is an eigen value of A^n.
7. State Cayley Hamilton theorem.
8. Define a linear transformation between two vector spaces.

P.T.O.

9. Prove that null space of a linear transformation from the vector space V_1 to V_2 is a subspace of V_1.

10. Find T (a, b) where the linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $T(1, 2) = (3, -1)$ and $T(0,1) = (2, 1)$. **(Weightage 6×1=6)**

Answer **any seven** from the following (Weightage **2 each**) :

11. Prove that every basis of a finite dimensional vector space V has the same number of elements.

12. Determine whether or not the vectors (1, 1, 2), (2, 3, 1) and (4, 5, 5) in $\mathbb{R}^3$ are linearly dependent.

13. Show that the equations $x + y + z = 6$, $3x + y + z = 8$, $-x + y - 2z = -5$, $-2x + 2y - 3z = -7$ are consistent and solve them.

14. Find all the non-trivial solutions of $2x - y + 3z = 0$; $3x + 2y + z = 0$; $x - 4y + 5z = 0$.

15. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

16. Prove that eigen values of a Hermitian matrix are all real.

17. Let $T : U \to V$ be a 1-1 linear map. Prove that the subset $\{u_1, u_2, ..., u_n\}$ is linearly independent if and only if $\{T(u_1), T(u_2), ..., T(u_n)\}$ is linearly independent.

18. Find the null space, range and their dimensions of the linear transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + y, z, x - z)$.

19. Let T be a linear operator on P_2, the set of all polynomials of degree ≤ 2, defined by $T(a_0 + a_1x + a_2x^2) = a_0 + (a_1 - a_2)x + (a_0 + a_1 + a_2)x^2$. Find T^{-1}.

20. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$. **(Weightage 7×2=14)**

Answer **any three** from the following (Weightage **3 each**) :

21. Let $\mathbb{R}^+$ be the set of all positive real numbers. Check whether $\mathbb{R}^+$ is a vector space over $\mathbb{R}$ or not with respect to the operations defined as $u + v = uv$ and $\alpha u = u^{\alpha}$ for all $u, v \in \mathbb{R}^+$ and $\alpha \in \mathbb{R}$.

22. Using row elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}.$$

23. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.

24. Diagonalise the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

25. Let $T : R^3 \to R^2$ be a linear transformation defined by $T(x, y, z) = (x + y, x + z)$. Find the matrix representation of T with respect to the ordered basis $X = \{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}$ in R^3 and $Y = \{(1, 0), (0,1)\}$ in R^2.

(Weightage 3×3=9)

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