



M 6054

Reg. No. : .....

Name : .....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.)

Examination, May 2014

CORE COURSE IN MATHEMATICS

6B13 MAT : Integral Transforms

Time : 3 Hours

Max. Weightage : 30

1. Fill in the blanks :

a) Laplace transform of  $\sin at$  is \_\_\_\_\_

b) Period of  $\cos x$  is \_\_\_\_\_

c) If  $f(x)$  is odd, then the value of  $\int_{-\pi}^{\pi} f(x) dx =$  \_\_\_\_\_

d)  $Z(a^n) =$  \_\_\_\_\_

(Weightage 1)

Answer **any six** from the following (weightage **1 each**) :

2. State the condition for the existence of Laplace transform.

3. Find  $L\left(\frac{\sin at}{t}\right)$ .

4. Find the inverse Laplace transform of  $\frac{1}{s^2+4s+5}$ .

5. Explain the half range Fourier sine series and cosine series expansion of functions.

6. State initial value theorem for Z-transform.

7. Find Z-transform of  $n^2$ .

8. Find Z-transform of  $\sin (t + T)$ .

9. Explain Fourier transform and inverse Fourier transform formulae.

10. Find Fourier cosine transform of  $f(x) = e^{-x}$ .

(Weightage  $6 \times 1 = 6$ )

Answer **any seven** from the following (weightage **2 each**) :

11. State and prove second shifting theorem for Laplace transform.

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12. Using convolution theorem, find  $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$ .
13. Find the Fourier series expansion of  $f(x) = |x|$  in the interval  $-\pi < x < \pi$ .
14. Express  $f(x) = e^x$  as a half range sine series in  $0 < x < 1$ .
15. Find the complex Fourier series of  $f(x) = \cos ax$ ,  $-\pi < x < \pi$ .
16. Find Z-transform of  $n(n-1) a^n u(n)$ .
17. Find the Z-transform of  $f * g$  where  $f(n) = u(n)$  and  $g(n) = 4^n u(n)$ .
18. Using partial fraction method, find the inverse Z-transform of  $\frac{z}{z^2 + 11z + 24}$ .
19. Find the Fourier sine integral of  $f(x) = e^{-kx}$ .
20. With usual notation, prove that

$$F_c \{f''(x)\} = -\omega^2 F_c \{f(x)\} - \sqrt{\frac{2}{\pi}} f'(0). \quad (\text{Weightage } 7 \times 2 = 14)$$

Answer **any three** from the following (weightage **3 each**):

21. State and prove Convolution theorem for Laplace transforms.
22. Using Laplace transform, solve the Laguerre's differential equation:

$$y'' + 2y' + 5y = e^{-t} \sin t, y(0) = 0, y'(0) = 1.$$

23. Obtain the Fourier series for the function  $f(x) = x + x^2$  in the interval  $(-\pi, \pi)$ . Deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

24. a) Using convolution method, find the inverse Z-transform of  $\frac{z^2}{(z-2)(z-3)}$ .
- b) Using power series method, find the inverse Z-transform of  $\log\left(\frac{z}{z+1}\right)$ .

25. Deduce Fourier integral representation formula from the Fourier series formula.

(Weightage  $3 \times 3 = 9$ )